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A Search-Theoretic Approach to Efficient Financial Intermediation

Fabien Tripier

Highlights

- The financial crisis has strengthened the role of non-financial deposits as a stable source of bank funding.
- Evidence suggests important switching costs on deposit markets.
- Financial intermediation is studied when households pay search costs to find adequate financial products and banks to attract depositors and to select borrowers.
- Financial intermediation is efficient when interest rates are posted by banks or bargained under a specific Hosios (1990) condition.
- Interbank market frictions are introduced to show how an crisis on this market leads to inefficient financial intermediation.

Abstract

This article develops a search-theoretic model of financial intermediation to study the efficiency condition of the banking sector. Competitive financial intermediation is determined by the search decisions of both households (to find adequate financial products) and banks (to attract depositors through marketing and to select borrowers through auditing) and by the interest rate setting mechanism. The efficiency of the competitive economy requires that interest rates are posted by banks or are bargained under a specific Hosios (1990) condition, which addresses the hold-up problem induced by search frictions on the credit and deposit markets. Interbank market frictions are introduced to show how an interbank market crisis leads to inefficient financial intermediation characterized by credit rationing and high net interest margin.

Keywords

Banking; Search, Matching, Switching Costs, Efficiency.

JEL

C78, D83, G21.

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RESEARCH AND EXPERTISE
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A Search-Theoretic Approach to Efficient Financial Intermediation [1]

Fabien Tripier*

1. Introduction

The financial crisis that began in 2007 has strengthened the role of non-financial deposits as a source of bank funding, which has become the new black.[2] Deposit funding is part of "the current "back to basics" policy" formulated by the ECB (2010)[3], and it goes back to the "back to basics" issue in finance: the efficiency of financial intermediation, which is the transformation

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[2]This expression is borrowed from a 2012 report of the company Ernst & Young for Australian banks entitled "The rise of the deposits", in which it is stated that "Deposits are the new black, lending playing second fiddle".

[3]This episode has been documented widely, notably by the ECB (2012): "bank funding strategies needed to be adjusted quickly in order to expand the customer deposit base and reduce the share of wholesale funding." Interestingly, the ECB (2010) makes a connection between the reversal from the interbank market to the retail deposit market and the crucial role played by bank marketing in the process: "As for other sources of funding, the crisis has resulted in an increased awareness of differences between banks, with banks with established brands gaining a competitive advantage vis-à-vis their weaker competitors."

of non-financial deposits into business loans by financial intermediaries. This paper revisits this issue. Its originality, which forms its contribution to the literature, is to assume that financial services to non-financial customers are characterized by relationship banking for both depositors and borrowers.[4] I provide empirical evidence that supports this assumption (see Section 2) and develop a search and matching model of financial intermediation based on this evidence. The optimality conditions are derived and the model is used to study the transmission of interbank market frictions to retail banking markets.

Efficient financial intermediation is a constrained-efficient equilibrium that is the social planner's solution to maximize steady-state welfare given search costs. From the social planner's point of view, the issue is to allocate an efficient amount of resources to search activities to ensure an efficient level of final good production. In a competitive economy, financial intermediation is determined by the search decisions of both households (to find adequate financial products) and banks (to attract depositors through marketing and to select borrowers through auditing). Even if markets are not frictionless, there are ways to reach efficiency. The first method[5] was demonstrated by Hosios (1990) and requires equality between the agent's bargaining power and the elasticity of the matching function with respect to its search effort. In this case, the Hosios (1990) condition is satisfied and the search externalities are internalized. However, this condition does not hold for financial intermediation because banks face search frictions in recruiting both depositors and borrowers. Agents decide, first, to search and then, if matched, bargain interest rates. When bargaining on interest rates, a bank considers its gains and losses regardless of whether the bargain with the non-financial agent succeeds. Without an agreement, the bank would lose not only the value of one financial relationship but also that of two relationships.

[4]These relationships are known in the literature as relationship banking, defined by Goddard et al. (2007) as follows: "One such topic is relationship banking, which can be defined simply as the provision of financial services repeatedly to the same customer". Relationship banking is not a new topic in financial intermediation. As explained below, the novelty of this paper is to consider relationship banking for both deposit and credit in a search model and to study the efficiency condition of financial intermediation.

[5]A second method, which relies on price posting and directed search, is studied thereafter.

Indeed, without an agreement with a depositor, the bank loses this depositor as well as a creditor who can no longer be financed. Likewise, without an agreement with a borrower, a bank loses this borrower as well as one depositor whose funds can no longer be invested. The bank faces a liquidity problem[6] which leads to the hold up problem. The hold up problem considered herein proceeds from the necessity of two investments in search and not of ex-ante investment in capital, technology or human capital as generally considered in the literature; e.g. Grout (1984), Malcomson (1997) and Acemoglu and Shimer (1999). The Hosios (1990) condition for efficiency is valid when there is no hold up problem. Otherwise, alternative bargaining powers are required to reach efficiency.[7] Contrary to the case studied by Acemoglu and Shimer (1999), efficiency can be achieved herein with ex-post barganing. The difficulty is that efficient bargaining powers are different from credit and deposit markets (even if matching functions are identical) and are functions of a large set of structural parameters (not only the elasticity parameter of the matching function). These properties of efficient bargaining powers hold whatever the negotiation is between two players (the bank plus one non-financial customers, a depositor or a borrower) or three players (the bank plus two non-financial customers, a depositor and a borrower).

Because efficient bargaining powers may be difficult to implement, I consider an alternative mechanism to set interest rates: price posting. If banks post interest rates on markets instead of bargaining over them with non-financial agents, the hold up problem identified herein would disappear. Banks jointly determine the search efforts and the interest rates and are not exposed to the double loss in the case of bargaining failure as described above. I show that the competitive economy is efficient if banks post financial contracts on the market and if non-financial agents direct their search toward banks. As in Acemuglu and Shimer (1999), the interest of price posting

[6]It is a liquidity problem because the bank cannot buy or sell assets (deposits or loans) quickly on financial markets. The crucial point here is that retail banking markets are sluggish as in Huang and Ratnovski (2011).

[7]Petrosky-Nadeau and Wasmer (2013) study the efficiency of financial intermediation with search on credit and labor markets. In their model, the Hosios (1990)'s condition is sufficient to guarantee efficiency because firms do not have the problem of liquidity considered here for banks.

is to avoid the hold up problem. This supplements its traditional interest in (single) market search models, thus making endogenous the Hosios (1990) condition; see Shimer (1996) and Moen (1997). I use this model of efficient financial intermediation to investigate the consequences of interbank market frictions in line with Duffie et al. (2005), Lagos and Rocheteau (2009), and Lagos et al. (2011). To provide a role for interbank market, banks are specialized either in the deposit activity or in the credit activity. Two matched banks negotiate a contract which specifies the mass of customers for each bank and a payment from the credit bank to the deposit bank. There is an exogenous probability of match dissolution on the interbank market that leads to the breakdown of relationships with non-financial customers. An increase in this probability, which can be interpreted as interbank market crisis, leads to an inefficient financial intermediation characterized by credit rationing and high net interest margin.[8]

The remainder of the paper is structured as follows. Section 2 gives the rationale for search frictions in the banking sector. The issue of financial intermediation and its socially optimal solution are presented in Section 3. The competitive equilibrium is defined in Section 4 when interest rates are ex-post bargained and in Section 5 when interest rates are posted. The discussion and concluding remarks are presented in Section 6.

2. Rationale For Search Frictions

The role of financial intermediation considered in the model developed herein does not follow from structural differences between deposits and loans[9], but from the existence of search frictions in financial markets. This Section gives the rationale for search frictions in the credit and deposit markets.

[8]The net interest margin is the difference between the credit interest rate and the deposit interest rate.

[9]The traditional role of financial intermediation is to transform assets, which are heterogeneous with respect to size, risk, or maturity. Here, there is a one-to-one correspondence between the per-period deposit of one household and the resources needed to finance a one period firm project.

2.1. Credit Market

Applying the search model to the credit market follows the literature initiated by Diamond (1990) and developed by Den Haan et al. (2003), Wasmer and Weil (2004), and Dell'Arricia and Garibaldi (2005). Two rationales for credit search frictions have been provided. The first rationale is based on the existence of long-term relationships between lenders and borrowers, which are known as lending relationships. Berger and Udell (1998) report an average duration of lending relationship between small business firms and commercial banks of 7.77 years. This was a robust observation in financial markets at the beginning of the lending relationship literature, which has been reviewed by Berger and Udell (1995) and Elyasiani and Goldberg (2004). Both Den Haan et al. (2003) and Wasmer and Weil (2004) invoke this literature to motivate their credit market search model.[10] The second rationale for credit search frictions is provided by Dell'Ariccia and Garibaldi (2005) and Craig and Haubrich (2013). They construct databases of credit flows from banks and show that the credit market in the United States is characterized by large, cyclical flows of credit expansion and contraction that may be explained in terms of the matching friction. Based on these rationales, numerous theoretical models incorporate the credit market search model to address macroeconomic and financial issues.[11]

2.2. Deposit Market

Applying the search model to the deposit market is a contribution of this paper to the literature on frictional financial markets.[12] Search frictions have already been considered on the credit

[10] Den Haan et al. (2003) argue that "... there is a matching friction in the market to establish entrepreneur-lender relationships. This friction highlights the importance of long-term relationships". They develop this argument in the Section "Motivation for matching friction" of their paper.

[11] See, among others, Beaubrun-Diant and Tripier (2013), Becsi et al. (2005, 2013), Chamley and Rochon (2001), Petrosky-Nadeau and Wasmer (2013), and Petrosky-Nadeau (2013).

[12] Another strand of literature considers the role of financial intermediaries in search-based models of monetary exchange *à la* Kiyotaki and Wright (1989) to explain the use of bank liabilities as a media of exchange, see He et al. (2005, 2008), and that banks improve welfare, see Berensten et al. (2007) and Gu et al. (2013).

market (as explained just above), on over-the-counter financial markets, first by Duffie et al. (2005) and then by Lagos and Rocheteau (2009) and Lagos et al. (2011), among others, but not on the deposit market.[13] The rationale for deposit market search frictions is, as for the credit market, the existence of long-term relationships between depositors and banks. I first document this fact, and I then explain it using the presence of switching costs for households and of relationship marketing by banks.

The European Commission (2009) published a survey on consumers' views regarding switching service providers to collect information about consumers' experiences switching providers and their ability to compare offers from various suppliers in several service sectors. The switching rate in the last two years is 11% for the banking industry as a whole, which is notably lower than the rates observed in other service sectors, such as car insurance (25%) or internet service (22%). The model developed in this paper does not apply to all financial services provided by banks, but to the remuneration of savings. It is therefore important to note that the switching rate for savings or investment products only remains low, approximately 13% against 9%, for current bank accounts. Furthermore, this low switching rate is not only observed in European countries. Kiser (2002) reports a mean relationship duration of 13.3 years from a survey of American consumers in Michigan.[14]

Switching costs is the most popular explanation for households' behavior on the deposit market. When a household decides to switch (or when she enters the market), she must spend time and resources to obtain information on services offered by banks; this is the search process for households on the deposit market. This search process would be costless and instantaneous without search frictions. However, the complexity of the retail financial market makes this search process costly and time-consuming. Indeed, Carlin (2009) argues that "[p]urchasing a

[13] Isoré (2012) also introduces search frictions in funding sources of banks, but with stakeholders and not with household depositors as considered here.

[14] Because the switching rate is a proxy for the (inverse of the) duration of customer relationships, it corresponds to a switching rate of approximately 7.5% per year or 15% every two years.

retail financial product requires effort. Because prices in the market are complex, consumers must pay a cost (time or money) to compare prices in the market." Similarly, according to Sirri and Tufano (1998), "[e]conomists acknowledge that consumers' purchasing decisions – whether for cars or funds – are complicated by the phenomenon of costly search". Accordingly, the European Commission (2009) reports that 43% of interviewed customers anticipated or experienced difficulties switching banking services and 37% think that it is very and/or fairly difficult to compare offers in the banking sector. Consistent with this evidence, I assume in the model developed herein that households pay search costs on the deposit market and that these search costs are interpreted as switching costs because a household must pay the costs to find another bank.[15]

Switching costs are intimately related to the practice of relationship marketing. The following observation about relationship marketing is made by Chiu et al. (2005): "marketing activities that attract, develop, maintain, and enhance customer relationships has changed the focus of a marketing orientation from attracting short-term, discrete transactional customers to retaining long-lasting, intimate customer relationships." Relationship marketing is therefore precisely devoted to increasing the cost of switching for customers. The underlying motivation of banks is to increase profits, as explained by Degryse and Ongena (2008): "Switching costs for bank customers represent an important source of rents for banks, and an important motive for the development of relationship (as opposed to transaction) banking." Sharpe (1997), Shy (2002), and Martin-Oliver et al. (2008) have established both theoretically and empirically the impact of switching costs on deposit interest rates using data for the United States, Finland, and Spain, respectively. In the model developed herein, bank search costs are interpreted as investment in relationship marketing because they are necessary to create long-term relationships with households.

[15] It is worth mentioning that if I identify both search costs and switching costs, Wilson (2012) develops a model devoted to distinguishing between search costs and switching costs.

3. The Issue of Financial Intermediation

This Section defines the issue of financial intermediation and presents the socially optimal solution.

3.1. Endowments and Technologies

I consider an economy with a raw good that cannot be consumed and a final (consumption) good that is produced by using the raw good as input. All agents (households, entrepreneurs, and banks) share the same linear utility function and discount factor for the future, denoted $\beta \in]0,1[$ with $\beta = 1/(1+r)$, where r is the associated interest rate. There are two production technologies with different qualities. Households possess the low-quality technology that produces $\rho^h > 0$ units of final good per unit of input. Entrepreneurs possess the high-quality technology that produces $z > \rho^h$ units of final good per unit of input. Entrepreneurs have better technology, but all raw goods are initially given to households. Each household holds an asset that delivers one unit of raw good per period. The economic issue is how to avoid autarky: how do we transfer raw goods from households to entrepreneurs without a market for the raw good (e.g., without direct finance)? This is the issue of financial intermediation, solved herein in the presence of search frictions.

3.2. Search Frictions

I first characterize search frictions on the credit market. Banks invest $\kappa^c v^c$ in the search to find entrepreneurs on the credit market, where κ^c is the search cost per unit of effort and v^c is the banks' search effort (assuming a unit continuum of banks, it is equal to the search effort of the representative bank). Search is costless for entrepreneurs, and the u^c unmatched entrepreneurs search for a bank.[16] The per-period flow of new lending relationships is given by the matching function $m^c(v^c, u^c)$, which has constant returns to scale and is increasing in both arguments.

[16] Search is exogenous for entrepreneurs and endogenous for households and banks.

The n^c matched entrepreneurs produce and remain matched with a probability $(1 - \delta^c)$, where $\delta^c \in]0, 1[$ is the probability of business failure.[17] The number of matched entrepreneurs evolves as follows

$$n^c_+ = (1 - \delta^c)\, n^c + m^c\,(u^c, v^c) \tag{1}$$

where the symbol $+$ is used to denote the next-period value of state variables. The population of entrepreneurs is set to $\overline{n}^c$ and satisfies $\overline{n}^c = n^c + u^c$.

Banks invest $\kappa^d v^d$ in the search to attract households to the deposit market, where κ^d is the search cost per unit of effort and v^d is the banks' search effort (assuming a unit continuum of banks, it is equal to the search effort of the representative bank). Unmatched households produce low-quality technology; a part, u^d, of them decide to search for a bank (and to pay the per-period cost κ^h), whereas another part, o^d, of the households prefer to remain outside the banking sector. The per-period flow of new deposit relationships is given by the matching function $m^d\left(v^d, u^d\right)$, which has constant returns to scale and is increasing in both arguments. The n^d matched households remain matched with a probability $\left(1 - \delta^d\right)$, where $\delta^d \in]0, 1[$ is a preference shock.[18] The number of matched households evolves as follows

$$n^d_+ = \left(1 - \delta^d\right) n^d + m^d\left(u^d, v^d\right) \tag{2}$$

The population of households is set to $\overline{n}^d$ and satisfies $\overline{n}^d = n^d + u^d + o^d$.

The number of productive entrepreneurs cannot exceed the number of depositors

$$n^c \leq n^d \tag{3}$$

where n^d is also the amount of deposits (each household deposits one indivisible unit of raw good) and n^c is the amount of credits (each entrepreneur borrows one indivisible unit of raw

[17] After a failure, the entrepreneur builds a new business project that should be audited by banks to be financed.

[18] The household decides to switch banks after a change in its demographic composition (e.g., births, divorce) or on the labor market (e.g., job loss, promotion). The household pays the search costs to find the relevant financial service given the new situation.

good). Finally, the matching technologies are Cobb-Douglas with the following properties

$$\begin{aligned} m^x(u^x, v^x) &= \overline{m}^x (v^x)^\varepsilon (u^x)^{1-\varepsilon} \\ q(\alpha^x) &= m^x(u^x, v^x)/v^x = \overline{m}^x (\alpha^x)^{\varepsilon-1} = p(\alpha^x)/\alpha^x \\ \partial m^x(u^x, v^x)/\partial u^x &= (1-\varepsilon)\, p(\alpha^x)\,,\ \partial m^x(u^x, v^x)/\partial v^x = \varepsilon q(\alpha^x) \end{aligned} \tag{4}$$

where $\alpha^x = v^x/u^x$ is the market tightness, $q(\alpha^x)$ and $p(\alpha^x)$ are the matching probabilities, for $x = \{c, d\}$ where c stands for credit and d for deposit. Without a loss of generality, the two matching functions share the same elasticity parameter ε, but the scale parameter $\overline{m}^x$ may be different.

Lemma 1 *The market tightness variables $\{\alpha^x\}_{x=\{c,d\}}$ determine the degree of financial intermediation and the social welfare.*

Proof. *See Appendix A.* ■

3.3. The Socially Optimal Solution

The socially optimal equilibrium is a constrained-efficient equilibrium. The social planner chooses search efforts to maximize steady–state welfare, taking the search frictions as given. The value function associated with the problem of the social planner is

$$\begin{aligned} &O\left(n^c, n^d\right) = \\ &\max_{u^d, \left\{n^x_+, v^x_+\right\}_{x=\{c,d\}}} \left\{n^c z + \left(\overline{n}^d - n^d - u^d\right)\rho^h + u^d\left(\rho^h - \kappa^h\right) - \kappa^d v^d - \kappa^c v^c + \beta O\left(n^c_+, n^d_+\right)\right\} \\ &-\lambda^c\left[n^c_+ - (1-\delta^c)\, n^c - m^c\left(\overline{n}^c - n^c, v^c\right)\right] \\ &-\lambda^d\left[n^d_+ - \left(1-\delta^d\right) n^d - m^d\left(u^d, v^d\right)\right] \\ &-\lambda^i\left(n^c - n^d\right) \end{aligned} \tag{5}$$

where the per-period utility flow is defined as the final goods produced by households and entrepreneurs less the search costs for households and banks. $\{\lambda^x\}_{x=\{c,d,i\}}$ are the Lagrangian multipliers associated with the constraints (1), (2), and (3). The next proposition presents the solution of (5).

Proposition 1 *The socially optimal equilibrium exists and is unique. Financial intermediation is socially optimal if the technology gap between households and entrepreneurs is sufficiently high.*

Proof. *The socially optimal allocation of resources is defined by the market tightness variables* $\{\alpha^x\}_{x=\{c,d\}}$ *that solve*

$$\alpha_o^d = \frac{\kappa^h}{\kappa^d}\frac{\varepsilon}{1-\varepsilon} \tag{6}$$

$$(r+\delta^c)\frac{\kappa^c}{\overline{m}^c}(\alpha_o^c)^{1-\varepsilon} + \left(r+\delta^d\right)\frac{\kappa^d}{\overline{m}^d}\left(\alpha_o^d\right)^{1-\varepsilon} = \varepsilon\left(z-\rho^h\right) - (1-\varepsilon)\kappa^c\alpha_o^c \tag{7}$$

The technology gap between households and entrepreneurs is $\left(z-\rho^h\right)$*, and it should be sufficiently large to ensure a positive value for* α_o^c*. See Appendix B for details.* ■

The socially optimal deposit market tightness α_o^d is given by equation (6) as a function of search costs and elasticity parameters of the matching functions.[19] To increase the amount of deposits, the social planner can increase the banks' search efforts, at the cost κ^d for the marginal productivity $m_2^d\left(u^d, v^d\right)$, or the households' participation, at the cost κ^h for the marginal productivity $m_1^d\left(u^d, v^d\right)$. Condition (6) makes equal the cost ratio, which is κ^h/κ^d, to the marginal productivity ratio, which is $m_1^d\left(u^d, v^d\right)/m_2^d\left(u^d, v^d\right) = \alpha^d(1-\varepsilon)/\varepsilon$, given the specification (4) of the matching function.

The socially optimal credit market tightness α_o^c is then given by equation (7), where α_o^d is given by (6). The LHS of (7) measures the matching costs of financial intermediation: how much

[19] This expression for the equilibrium tightness is common in search models with two endogenous participation rules, not one, as is usually assumed. Indeed, participation is endogenous for firms, but it is exogenous for workers in the standard labor market search model. Wasmer and Weil (2004) obtain an expression for the credit market tightness similar to (6) because they consider the endogenous participation of both entrepreneurs and bankers (but the exogenous participation of workers on the labor market). Here, participation is endogenous for banks on the deposit and credit markets, endogenous for households on the deposit market, and exogenous for entrepreneurs on the credit market.

it costs to collect one unit of deposit and to select one entrepreneur. The matching costs are equal to $(\kappa^x/\overline{m}^x)(\alpha_o^x)^{1-\varepsilon}$: the per-period search cost κ^x on market $x = \{c, d\}$ is divided by the probability of matching, $\overline{m}^x(\alpha_o^x)^{\varepsilon-1}$. Matching costs are discounted by $(r + \delta^x)$, the sum of the rate of time preference and of the separation probability. The RHS of (7) measures the social benefits of financial intermediation: the technology gap between households and entrepreneurs, weighted by ε, less the opportunity costs of being matched for the entrepreneur, weighted by $(1-\varepsilon)$. Outside the match, the entrepreneur would have contributed to social welfare by searching: with a marginal productivity $m_1^c(u^c, v^c)$, it would have created the value of a new match, which is equal to $\varepsilon \times \kappa^c/m_2^c(u^c, v^c)$. The last term of the RHS of (7) corresponds to the product $m_1^c(u^c, v^c) \times \varepsilon \times \kappa^c/m_2^c(u^c, v^c)$, given the specification (4) of the matching function.

4. Competitive Financial Intermediation with Interest Rate Bargaining

This Section presents the competitive equilibrium with ex-post bargaining and undirected search by non-financial agents.

4.1. Search

4.1.1. Non-Financial Agents

Households' value functions are denoted D^y, where $y = \{h, m, u\}$ refers to the household states: non-participating, matched, and searching, respectively. They are defined by

$$D^h = \rho^h + \beta D^h \tag{8}$$

$$D^m(\rho^d) = \rho^d + (1-\delta^d)\beta D^m(\rho^d) + \delta^d \beta D^u \tag{9}$$

$$D^u = \rho^h - \kappa^h + p(\alpha^d)\beta D^m(\rho^d) + \left[1 - p(\alpha^d)\right]\beta D^u \tag{10}$$

If the household does not participate, she produces ρ^h of final goods. If she decides to search for a bank, she still produces ρ^h but now pays κ^h as a search cost and has a probability $p(\alpha^d)$

of forming a match with a bank. When she is matched, the household receives ρ^d units of the final good as deposit interests and remains in this state with a probability $\left(1-\delta^d\right)$. Unmatched households decide whether to search. The free entry condition on the deposit market implies $D^h = D^u$ or, equivalently,

$$\begin{aligned} D^h = D^u \rightarrow \quad \kappa^h = & \quad p\left(\alpha^d\right)\beta\left[D^m\left(\rho^d\right) - D^u\right] \\ \kappa^h = & \quad p\left(\alpha^d\right)\frac{\rho^d - \rho^h}{r+\delta^d} \end{aligned} \tag{11}$$

given (8), (9), and (10). The entry of households is such that the search cost, κ^h, is equal to search payoff: with a probability $p\left(\alpha^d\right)$, the household earns the difference between deposit interests and home production $\left(\rho^d - \rho^h\right)$ discounted by $\left(r+\delta^d\right)$.

Entrepreneurs' value functions are denoted as C^y, where $y = \{m, u\}$ refers to the entrepreneur states: matched and unmatched, respectively. They are defined by

$$C^u = p\left(\alpha^c\right)\beta C^m\left(\rho^c\right) + \left(1 - p\left(\alpha^c\right)\right)\beta C^u \tag{12}$$

$$C^m\left(\rho^c\right) = z - \rho^c + \left(1-\delta^c\right)\beta C^m\left(\rho^c\right) + \delta^c\beta C^u \tag{13}$$

The per-period utility is zero when entrepreneurs search and $(z - \rho^c)$ when matched, where ρ^c is the amount of credit interests. The transition probabilities between states are $p\left(\alpha^c\right)$ and δ^c.

4.1.2. Bank Search Efforts

The representative bank maximizes the discounted sum of profits (defined as the credit interests less both the deposit interests and the search costs) subject to the constraints (1), (2), and (3). The bank value function is

$$\begin{aligned} B\left(n^c, n^d\right) \quad = \quad & \max_{\left\{n_+^x, v^x\right\}_{x=c,d}} \left\{\rho^c n^c - \rho^d n^d - \kappa^d v^d - \kappa^c v^c + \beta B\left(n_+^c, n_+^d\right)\right\} \\ & -\lambda^c\left[n_+^c - \left(1-\delta^c\right)n^c - q\left(\alpha^c\right)v^c\right] \\ & -\lambda^d\left[n_+^d - \left(1-\delta^d\right)n^d - q\left(\alpha^d\right)v^d\right] \\ & -\lambda^i\left(n^c - n^d\right) \end{aligned} \tag{14}$$

The bank chooses search efforts $\{v^x\}_{x=\{c,d\}}$ given the interest rates $\{\rho^x\}_{x=\{c,d\}}$ such that

$$(r+\delta^c)\,\frac{\kappa^c}{\overline{m}^c}\,(\alpha^c)^{1-\varepsilon}+\left(r+\delta^d\right)\frac{\kappa^d}{\overline{m}^d}\left(\alpha^d\right)^{1-\varepsilon}=\rho^c-\rho^d \tag{15}$$

see Appendix C.1 for details. The optimality condition for banks, namely (15), can be compared with its counterpart for the social planner, namely (7). The LHS terms of (7) and (15) are identical and correspond to the matching costs of financial intermediation. The RHS terms correspond to benefits of financial intermediation, which may differ. The bank's benefits are the interest margin, i.e., the difference between credit and deposit interests in (15), which may not coincide with the social benefits in (7).

4.2. Bargaining

The bank is matched with n^d depositors and n^c borrowers and therefore bargains with $\left(n^d+n^c\right)$ customers. Bargaining is individual and ex-post. When the bank bargains with a customer, she is considered as the marginal customer assuming that all other negotiations are terminated. Under this assumption, there are only two players in the bargaining process, namely the bank and the marginal customer and the Nash Solution can be viewed as the outcome of a bilateral bargaining game in the strategic approach; see Rubinstein (1982) and Binmore et al. (1986). The case of multilateral bargaining game is discussed in Section 4.4. Interest rates satisfy

$$\rho^x=\arg\max\;\left(X^m-X^u\right)^{1-\eta^x}\left(\Delta B^x\right)^{\eta} \tag{16}$$

for $x=\{c,d\}$ and $X=\{C,D\}$. The parameter $(1-\eta^x)$ measures the bargaining power of banks, and η^x measures the bargaining power of non-financial customers for $x=\{c,d\}$. The surplus of non-financial customers (X^m-X^u) can be directly computed using the value function definitions (9) and (10) for households and (12) and (13) for entrepreneurs. It is less direct for the bank's surplus ΔB^x, given the constraint (3).

Consider first the case where the marginal customer is a depositor. If bargaining fails, the bank is deprived of one depositor and one borrower cannot produce because the bank cannot provide the raw good, which is necessary for the entrepreneur to produce. I assume that if the production process is interrupted, the entrepreneur should be audited once again to restart the production process. The same occurs for borrowers. If bargaining fails, the bank is deprived of one borrower and one depositor because the bank cannot transform the raw good into a final good to pay the deposit interest. In fact, the depositor withdraws her unit of raw good if the utility in the unmatched state is higher than the utility in the matched state with no deposit interests, that is: $D^u > D^m\left(\rho^d\right)\big|_{\rho^d=0}$. In this case, the depositor leaves the bank, consumes the final good and prospects for another bank. Using the value function definitions (9) and (10), it occurs when

$$\rho^h + \beta D^h > \left(1-\delta^d\right)\beta D^m\left(\rho^d\right) + \delta^d \beta D^u \tag{17}$$

or, equivalently, $\rho^h > \left(1-\delta^d\right)\kappa^h/p\left(\alpha^d\right)$ given the free entry condition on the deposit market (11). Hereafter, I assume that the condition (17) holds.

If the bargaining with a customer fails, the bank loses two customers and not only one. This situation results from the existence of search frictions on the two markets and leads to the hold up problem.To formalize this point, I introduce the functions $n_1^y\left(n^x\right)$ which satisfy

$$n_1^y\left(n^x\right) = \begin{cases} 0 & \text{no hold up problem} \\ 1 & \text{hold up problem} \end{cases} \tag{18}$$

for $x = \{c, d\}$, $y = \{c, d\}$ and $y \neq x$. The loss of one depositor implies the loss of one borrower if $n_1^c\left(n^d\right) = 1$ and not otherwise. Similarly, the loss of one borrower implies the loss of one depositor if $n_1^d\left(n^c\right) = 1$ and not otherwise. Finally, the bank surplus is as follows

$$\begin{aligned} \Delta B^x &= \left.\frac{\partial B\left(n^c, n^d\right)}{\partial n^x}\right|_{n^y = n^y\left(n^x\right)} \\ &= \rho^c - \rho^d + \left(1-\delta^c\right)\frac{\kappa^c}{q\left(\alpha^c\right)} + \left(1-\delta^d\right)\frac{\kappa^d}{q\left(\alpha^d\right)} - \left[1 - n_1^y\left(n^x\right)\right]\left(1+r\right)\frac{\kappa^x}{q\left(\alpha^x\right)} \end{aligned} \tag{19}$$

for $x = \{c, d\}$, $y = \{c, d\}$ and $y \neq x$; see the Appendix C.2 for details. The bank's surplus on the market x is equal to the net interest margin plus the value of financial relationships on the two markets, if these relationships are not destroyed by the probability $(1-\delta^x)$, less the discounted matching costs. If $n_1^y(n^x) = 1$, there is a hold up problem with the non-financial customer x and the bank cannot subtract the matching cost from its surplus. If $n_1^y(n^x) = 0$, there is no hold up problem and the bank's surplus is lower than if $n_1^y(n^x) = 1$. The hold up problem affects the bank's surplus and, therefore, impact the interest rates and search decisions, as shown in the next Section.

4.3. Competitive Equilibrium

I define the competitive equilibrium and then discuss its normative properties.

Definition 1 *The competitive financial intermediation with bargained interest rates is characterized by the interest rates $\{\rho_b^x\}_{x=\{c,d\}}$ which satisfy*

$$\rho_b^d = \rho^h - \left(1-\delta^d\right)\frac{\kappa^h}{\overline{m}^d\left(\alpha_b^d\right)^{\varepsilon}} + \left(\frac{\eta^d}{1-\eta^d}\right)(1+r)\left[\frac{\kappa^d}{\overline{m}^d}\left(\alpha_b^d\right)^{1-\varepsilon} + n_1^c\left(n^d\right)\frac{\kappa^c}{\overline{m}^c}\left(\alpha_b^c\right)^{1-\varepsilon}\right] \tag{20}$$

$$\rho_b^c = z - \left(r+\delta^c+p\left(\alpha_b^c\right)\right)\left(\frac{\eta^c}{1-\eta^c}\right)\left[\frac{\kappa^c}{\overline{m}^c}\left(\alpha_b^c\right)^{1-\varepsilon} + n_1^d\left(n^c\right)\frac{\kappa^d}{\overline{m}^d}\left(\alpha_b^d\right)^{1-\varepsilon}\right] \tag{21}$$

where the equilibrium market tightness variables $\{\alpha_b^x\}_{x=\{c,d\}}$ are the solution of

$$\frac{\kappa^h}{\overline{m}^d\left(\alpha_b^d\right)^{\varepsilon}} = \left(\frac{\eta^d}{1-\eta^d}\right)\left[\frac{\kappa^d}{\overline{m}^d}\left(\alpha_b^d\right)^{1-\varepsilon} + n_1^c\left(n_b^d\right)\frac{\kappa^c}{\overline{m}^c}\left(\alpha_b^c\right)^{1-\varepsilon}\right] \tag{22}$$

$$\begin{aligned} & \left(r+\delta^c\right)\frac{\kappa^c}{\overline{m}^c}\left(\alpha_b^c\right)^{1-\varepsilon} + \left(r+\delta^d\right)\left(\frac{1-\eta^c}{1-\eta^d}\right)\frac{\kappa^d}{\overline{m}^d}\left(\alpha_b^d\right)^{1-\varepsilon} \\ = \; & \left(1-\eta^c\right)\left(z-\rho^h\right) - \eta^c\kappa^c\alpha_b^c - \left(r+\delta^c+p\left(\alpha_b^c\right)\right)\eta^c n_1^d\left(n_b^c\right)\frac{\kappa^d}{\overline{m}^d}\left(\alpha_b^d\right)^{1-\varepsilon} \\ & - \left(r+\delta^d\right)\left(1-\eta^c\right)\left(\frac{\eta^d}{1-\eta^d}\right)n_1^c\left(n_b^d\right)\frac{\kappa^c}{\overline{m}^c}\left(\alpha_b^c\right)^{1-\varepsilon} \end{aligned} \tag{23}$$

Appendix C.3 provides the resolution details for the Nash bargaining process, and Appendix C.4 shows how to obtain the equilibrium conditions for market tightness.

The equilibrium deposit interest rate given by (20) is equal to (i) the value of self-production, ρ^h, (ii) less the household value of the financial relationship, $\kappa^h/p\left(\alpha^d\right)$, which is preserved by the household with the probability $\left(1-\delta^{d}\right)$, (iii) plus the share $\eta^{d}/\left(1-\eta^{d}\right)$ of the bank values of financial relationships, $\kappa^{x}/q\left(\alpha^{x}\right)$, discounted by $(1+r)$ for $x=\{c,d\}$. When $n_{1}^{c}\left(n^{d}\right)$ is equal to zero, the household receives only a share of the bank value of the financial relationship on the deposit market, as is commonly the case in models that use search frictions. Here, the novelty is that when $n_{1}^{c}\left(n^{d}\right)$ is equal to unity, the household also receives a share of the bank value of the financial relationship on the credit market. This is the hold up problem: because the bank is subject to search frictions on the credit market, the household succeeds in appropriating a share of the lending relationship's value, even if the household does not participate in the search externalities on the credit market. All other things being equal, the hold up problem makes deposit interests higher.

The equilibrium credit interest rate given by (21) is equal to (i) the value of business production, z, (ii) less the share $\eta^{c}/\left(1-\eta^{c}\right)$ of the bank value of financial relationships, $\kappa^{x}/q\left(\alpha^{x}\right)$, discounted by[20] $\left(r+\delta^{c}+p\left(\alpha_{b}^{c}\right)\right)$ for $x=\{c,d\}$. When $n_{1}^{d}\left(n^{c}\right)$ is equal to zero, the entrepreneur receives only a share of the bank value of the financial relationship on the credit market, as is commonly the case in models that use search frictions. As previously, when $n_{1}^{d}\left(n^{c}\right)$ is equal to unity, the hold up problem occurs: because the bank is subject to search frictions on the deposit market, the entrepreneur succeeds in appropriating a share of the deposit relationship's value even if the entrepreneur does not participate in the search externalities on the deposit market. All other things being equal, the hold up problem makes credit interests lower.

Equations (22) and (23) show how the hold up problem influences the equilibrium values of the tightness variables. When households appropriate a share of the lending relationship's value, i.e., $n_{1}^{c}\left(n_{b}^{d}\right)=1$ in the RHS term of equation (22), they are willing to pay a higher matching cost, which corresponds to the LHS term of equation (22). For a given value of the credit market tightness α_{b}^{c}, this mechanism tends toward a fall in the deposit market tightness α_{b}^{c}.

[20] The discount rate can also be written as $\left[(1+r)+p\left(\alpha^{c}\right)-\left(1-\delta^{c}\right)\right]$. It is composed of the discount rate for time preference, namely $(1+r)$, and of the difference in probabilities of being matched according to the current state, namely, $p\left(\alpha^{c}\right)$, if unmatched, and $\left(1-\delta^{c}\right)$, if matched.

For banks, the hold up problem lowers the payoff of financial intermediation, as in the RHS term of equation (23) for $n_1^d\left(n_b^c\right)=n_1^c\left(n_b^d\right)=1$; consequently, the matching costs of financial intermediation should decrease, i.e., the LHS term of equation (23). For a given value of the deposit market tightness α_b^d, this mechanism acts toward a fall in the credit market tightness α_b^c.

The next proposition shows how the bargaining process can offset the effects of the hold up problem.

Proposition 2 *There exist values for bargaining power that make the competitive financial intermediation efficient.*

Proof. *The values of bargaining powers* $\{\eta_o^x\}_{x=c,d}$ *imply that* $\{\alpha_b^x=\alpha_o^x\}_{x=c,d}$ *where* $\{\alpha_o^x\}_{x=c,d}$ *solve (6) and (7) and* $\{\alpha_b^x\}_{x=c,d}$ *solve (22) and (23). They are*

$$\eta_o^d=(1-\varepsilon)\left[1+\varepsilon n_1^c\left(n_b^d\right)\frac{\kappa^c}{\kappa^d}\frac{q\left(\alpha_o^d\right)}{q\left(\alpha_o^c\right)}\right]^{-1}\leq(1-\varepsilon) \tag{24}$$

$$\eta_o^c=(1-\varepsilon) \tag{25}$$

$$\times\frac{z-\rho^h+\kappa^c\alpha_o^c-\left(r+\delta^d\right)\frac{\frac{\kappa^d}{q\left(\alpha_o^d\right)}+n_1^c\left(n_b^d\right)\frac{\kappa^c}{q\left(\alpha_o^c\right)}}{\varepsilon+\varepsilon n_1^c\left(n_b^d\right)\frac{\kappa^c}{\kappa^d}\frac{q\left(\alpha_o^d\right)}{q\left(\alpha_o^c\right)}}}{z-\rho^h+\kappa^c\alpha_o^c-\left(r+\delta^d\right)\frac{\frac{\kappa^d}{q\left(\alpha_o^d\right)}+n_1^c\left(n_b^d\right)\frac{\kappa^c}{q\left(\alpha_o^c\right)}}{\varepsilon+\varepsilon n_1^c\left(n_b^d\right)\frac{\kappa^c}{\kappa^d}\frac{q\left(\alpha_o^d\right)}{q\left(\alpha_o^c\right)}}+\left(r+\delta^c+p\left(\alpha_o^c\right)\right)n_1^d\left(n_b^c\right)\frac{\kappa^d}{q\left(\alpha_o^d\right)}}\leq(1-\varepsilon)$$

where the socially optimal values for market tightness $\{\alpha_o^x\}_{x=c,d}$ *do not depend on the bargaining power. See Appendix C.5 for details.* ■

Equations (24) and (25) generalize the Hosios (1990) condition for efficiency. In standard search models, the Hosios (1990) condition imposes equality between the agent's bargaining power and the elasticity of the matching function with respect to its search effort. In this case, the search externalities are internalized. This condition would have been sufficient in the search model of financial intermediation proposed in this paper without the hold problem: the conditions (24) and (25) reduce to

$$\eta_o^d = \eta_o^c = (1 - \varepsilon) \tag{26}$$

for $n_1^c\left(n_b^d\right) = n_1^d\left(n_b^c\right) = 0$. With the hold up problem, the Hosios (1990) condition is no longer sufficient to ensure efficiency. The second terms in equations (24) and (25), which multiply the elasticity coefficient $(1 - \varepsilon)$, generalize the Hosios (1990) condition to address the hold up problem. The next corollary explains the correction.

Corollary 1 *Efficiency of financial intermediation requires that the banks' bargaining power increase when the hold up problem occurs. Otherwise, the deposit market tightness is lower than the socially optimal level and credit rationing may occur.*

Proof. *It follows from proposition 2, see Appendix C.6 for details.* ■

If the bargaining power values are fixed to the Hosios (1990) values given by the equation (26), inefficient financial intermediation occurs because of the hold up problem. In Appendix C.6, I demonstrate that the competitive deposit market tightness is equal to or below its socially optimal level. The hold-up problem stimulates the entry of households into the deposit market who are willing to pay higher matching costs. The competitive credit market tightness can be lower than, equal to or higher than its socially optimal level because of the existence of two effects. First, the hold up by entrepreneurs makes the competitive credit market tightness lower because banks reduce their search efforts in response to a cut in credit interests. Second, low tightness on the deposit market reduces the matching costs of banks on this market, which are therefore willing to pay higher matching costs on the credit market. Remember that banks consider the total matching costs to be the sum of the matching costs on the deposit market and on the credit market, see the LHS of (15). For a given net interest margin, e.g., the RHS of (15), smaller matching costs on one market imply higher matching costs on the other one at equilibrium. If the first effect dominates the second, inefficient financial intermediation occurs with excessive credit rationing; otherwise, it occurs with an excessive investment of scarce resources in the financial sector.

4.4. The Case of Multilateral Bargaining

In the case of bilateral bargaining, all other negotiations are assumed to be terminated. However, because of the hold up problem, this assumption may not be considered appropriate. When a customer threatens to leave the bank, the other customer, which risks losing her financial relationship, may be interested in renegotiating with the bank. In this case, the bargaining is multilateral. Chae and Yang (1992), Krishna and Serrano (1996), and Suh and Wen (2006) show how reconciling the axiomatic and strategic approaches to multilateral bargaining problem.[21] The Nash solution for three agents should be applied as follows

$$\left\{\rho^d, \rho^c\right\} = \arg\max \left\{(D^m - D^u)^{\eta^d} (C^m - C^u)^{\eta^c} (\Delta B)^{\eta^b}\right\} \tag{27}$$

where $\eta^b = 1 - \eta^c - \eta^d$. The bargaining problem is resolved in Section D. I show the necessary conditions on the bargaining powers $\left\{\eta^d, \eta^c\right\}$ to ensure the efficiency of the financial intermediation. Once again, as in the case of bilateral bargaining, the bargaining powers differ from the standard Hosios condition (26) and are complex functions of the structural parameters. Moreover, bargaining powers should be different for the deposit and the borrower, $\eta^c \neq \eta^d$, to ensure efficiency - see Section D. This result has important implications for economic foundations of (27). For example, in the Krishna and Serrano (1996) game with three players, the bargaining power is $1/(1 + 2\omega)$ for the first proposer and $\omega/(1 + 2\omega)$ for the two responders where ω is the degree of impatience – see also Serrano (2008). If the bank is the first proposer and the non-financial agents the two responders, it implies: $\eta^b = 1/(1 + 2\omega)$ and $\eta^c = \eta^d = \omega/(1 + 2\omega) = \omega\eta^b$. In this case, there is no value for ω that ensures the efficiency of the competitive financial intermediation when bargaining is multilateral.

[21] See Suton (1986) for an exposition of the difficulty of extending the two-players Rubinstein (1982) model to n-players game.

5. Competitive Financial Intermediation With Interest Rate Posting

This Section presents the competitive equilibrium when banks post interest rates and non-financial agents direct their search toward banks. The resolution of the search model of financial intermediation with price posting is inspired by that of Kaas and Kircher (2013) developed for the labor market with large firms.

5.1. Directed Search

5.1.1. Households

The value function associated with the non-participating state is unchanged and given by (8). The value function associated with the searching state becomes

$$D^u = \rho^h - \kappa^h + p\left(\alpha^d\left(\rho^d_+\right)\right)\beta D^m\left(\rho^d_+\right) + \left[1 - p\left(\alpha^d\left(\rho^d_+\right)\right)\right]\beta D^u \tag{28}$$

where $D^m\left(\rho^d_+\right)$ is the value function associated with the matched state (9) for the posted interest rate ρ^d_+. Assuming that two interest rates are posted on the deposit market $\left\{\rho^d_+, \overline{\rho}^d_+\right\}$, $\overline{\rho}^d_+$ being the equilibrium rate and ρ^d_+ the rate posted by a bank that deviates from the equilibrium value. Households can search for a bank that offers the equilibrium rate or for the deviating bank. At the equilibrium, the search payoffs should be equal or, equivalently, the value function (28) is the same for ρ^d_+ and $\overline{\rho}^d_+$

$$\begin{aligned} & \rho^h - \kappa^h + p\left(\alpha^d\left(\rho^d_+\right)\right)\beta D^m\left(\rho^d_+\right) + \left[1 - p\left(\alpha^d\left(\rho^d_+\right)\right)\right]\beta D^u \\ = \; & \rho^h - \kappa^h + p\left(\alpha^d\left(\overline{\rho}^d_+\right)\right)\beta D^m\left(\overline{\rho}^d_+\right) + \left[1 - p\left(\alpha^d\left(\overline{\rho}^d_+\right)\right)\right]\beta D^u \end{aligned} \tag{29}$$

Simplifications give

$$p\left(\alpha^d\left(\rho^d_+\right)\right)\beta\left[D^m\left(\rho^d_+\right) - D^u\right] = p\left(\alpha^d\left(\overline{\rho}^d_+\right)\right)\beta\left[D^m\left(\overline{\rho}^d_+\right) - D^u\right] \tag{30}$$

I show in Appendix E.1 how to use the no-arbitrage condition defined by equation (30) to get the elasticity of the bank matching probability with respect to the deposit interest rate

$$\frac{\partial q\left(\alpha^d_+\left(\rho^d_+\right)\right)}{\partial \rho^d_+}\frac{\rho^d_+}{q\left(\alpha^d\left(\rho^d_+\right)\right)} = \left(\frac{1-\varepsilon}{\varepsilon}\right)\left(\frac{\rho^d_+}{\rho^d_+ - \rho^h}\right) > 0 \tag{31}$$

This expression will be necessary to determine the bank's pricing strategy. The sign of the partial derivative is positive: if the deviating bank increases its posted interest rates, more households search toward this bank. Therefore, the probability to find new depositors for this bank is higher even if its search effort (v^d) is constant.

5.1.2. Entrepreneurs

The value function associated with the matched state is defined by (13). The value function associated with the searching state becomes

$$C^u = p\left(\alpha^c\left(\rho_+^c\right)\right)\beta C^m\left(\rho_+^c\right) + \left(1 - p\left(\alpha^c\left(\rho_+^c\right)\right)\right)\beta C^u \tag{32}$$

Assuming that two interest rates are posted on the market $\left\{\rho_+^c, \overline{\rho}_+^c\right\}$, $\overline{\rho}_+^c$ being the equilibrium rate and ρ_+^c the rate posted by a bank that deviates from the equilibrium value. Entrepreneurs can search for a bank that offers the equilibrium rate or for the deviating bank. At the equilibrium, the search payoffs should be equal or, equivalently, the value function (32) is the same for ρ_+^c and $\overline{\rho}_+^c$

$$p\left(\alpha^c\left(\rho_+^c\right)\right)\beta C^m\left(\rho_+^c\right) + \left(1 - p\left(\alpha^c\left(\rho_+^c\right)\right)\right)\beta C_+^u \tag{33}$$

$$= \quad p\left(\alpha^c\left(\overline{\rho}_+^c\right)\right)\beta C^m\left(\overline{\rho}_+^c\right) + \left(1 - p\left(\alpha^c\left(\overline{\rho}_+^c\right)\right)\right)\beta C_+^u \tag{34}$$

Simplifications give

$$p\left(\alpha^c\left(\rho_+^c\right)\right) = p\left(\alpha^c\left(\overline{\rho}_+^c\right)\right)\frac{C^m\left(\overline{\rho}_+^c\right) - C^u}{C^m\left(\rho_+^c\right) - C^u} \tag{35}$$

I show in Appendix E.2 how to use the no-arbitrage condition defined by equation (35) to get the elasticity of the bank matching probability with respect to the credit interest rate

$$\frac{\partial q\left(\alpha^c\left(\rho_+^c\right)\right)}{\partial \rho_+^c}\frac{\rho_+^c}{q\left(\alpha^c\left(\rho_+^c\right)\right)} = -\left(\frac{1-\varepsilon}{\varepsilon}\right)\frac{1-\left[1-\delta^c - p\left(\alpha^c\left(\rho_+^c\right)\right)\right]\beta}{1-\left(1-\delta^c\right)\beta}\left(\frac{\rho_+^c}{z-\rho_+^c}\right) < 0 \tag{36}$$

This expression will be necessary to determine the bank's pricing strategy. The sign of the partial derivative is negative: if a bank decreases its posted interest rates, more entrepreneurs search toward this deviating bank. Therefore, the probability to find new borrowers for this bank is higher even if its search effort (v^c) is constant.

5.2. Interest Rate Posting

The state variable ϱ_+^x for the market $x = \{c, d\}$ measures the amount of interests received or paid by the bank, which evolves as follows

$$\varrho_+^x = (1 - \delta^x)\,\varrho^x + q\left(\rho_+^x\right) v^x \rho_+^x, \text{ for } x = \{d, c\} \tag{37}$$

This variable is a state variable because the bank cannot revise interest rates posted in the past. The representative bank maximizes

$$\begin{aligned}
& P\left(n^c, n^d, \varrho^c, \varrho^d\right) & (38)\\
= & \max_{\left\{\varrho_+^x, n_+^x, v^x, \rho_+^x\right\}_{x=\{d,c\}}} \left\{\varrho^c - \varrho^d - \kappa^d v^d - \kappa^c v^c + \beta P\left(n_+^c, n_+^d, \varrho_+^c, \varrho_+^d\right)\right\} \\
& -\lambda^c\left[n_+^c - (1-\delta^c)\,n^c - q\left(\alpha^c\left(\rho_+^c\right)\right) v^c\right] - \lambda^d\left[n_+^d - \left(1-\delta^d\right) n^d - q\left(\alpha^d\left(\rho_+^d\right)\right) v^d\right] \\
& -\mu^d\left[\varrho_+^d - \left(1-\delta^d\right)\varrho^d - q\left(\alpha^d\left(\rho_+^d\right)\right) v^d \rho_+^d\right] - \mu^c\left[\varrho_+^c - (1-\delta^c)\,\varrho^c - q\left(\alpha^c\left(\rho_+^c\right)\right) v^c \rho_+^c\right] \\
& -\lambda^i\left(n^c - n^d\right)
\end{aligned}$$

The price posting strategy is determined by the first order condition of program associated with the posted interest rate ρ_+^x on the market $x = \{d, c\}$:

$$\rho_+^x : \lambda^x \frac{\partial q\left(\alpha^x\left(\rho_+^x\right)\right)}{\partial \rho_+^x} v^x + \mu^x \left[\frac{\partial q\left(\alpha^x\left(\rho_+^x\right)\right)}{\partial \rho_+^x}\rho_+^x + q\left(\alpha^x\left(\rho_+^x\right)\right)\right] v^x = 0,\ x = \{c, d\} \tag{39}$$

see Section E.3. The first term account for the impact of ρ_+^x on the creation of new financial relationship, which values are λ^x, according to $q_1\left(\alpha^x\left(\rho_+^x\right)\right) v^x$, namely the reaction of the matching probability times the search effort, v^x. The sign of $q_1\left(\alpha^x\left(\rho_+^x\right)\right)$ depends on the market x as explained in Section 5.1. The second term account for the impact of ρ_+^x on the variation of the value function induced by the new amount of interests, which is equal to μ^x the derivative of the value function with respect to ϱ_+^d. Varying ρ_+^x impacts directly the amount of interests for the flow of new customers, $q(\cdot)\,v^x$, and indirectly because of the variation in this flow, $q_1(\cdot)\,\rho_+^x v^x$. The equilibrium value of the two multipliers are $\mu^c = 1/\left(r + \delta^c\right)$ and $\mu^d = -1/\left(r + \delta^d\right)$. The sign of μ^c is positive and that of μ^d negative, because credit interests increase the value function whereas deposit interests lower it. Variation in interests are discounted at the rate $(r+\delta^x)$ that takes into account the preference for the present and duration of the commitment for posted interest rates.

5.3. Competitive Equilibrium

I define the competitive equilibrium and then discuss its normative properties.

Definition 2 *The competitive financial intermediation with posted interest rates is characterized by the interest rates* $\left\{\rho_p^x\right\}_{x=\{c,d\}}$ *that satisfy*

$$\rho_p^d = \rho^h + \left(\frac{1-\varepsilon}{\varepsilon}\right)\left(r+\delta^d\right)\frac{\kappa^d}{\overline{m}^d}\left(\alpha_p^d\right)^{1-\varepsilon} \tag{40}$$

$$\rho_p^c = z - \left(\frac{1-\varepsilon}{\varepsilon}\right)\left[\left(r+\delta^c\right)\frac{\kappa^c}{\overline{m}^c}\left(\alpha_p^c\right)^{1-\varepsilon} + \kappa^c\alpha_p^c\right] \tag{41}$$

where the equilibrium market tightness variables $\left\{\alpha_p^x\right\}_{x=\{c,d\}}$ *are the solution of*

$$\alpha_p^d = \frac{\kappa^h}{\kappa^d}\left(\frac{\varepsilon}{1-\varepsilon}\right) \tag{42}$$

$$\left(r+\delta^c\right)\frac{\kappa^c}{\overline{m}^c}\left(\alpha_p^c\right)^{1-\varepsilon}\frac{\kappa^c}{q\left(\alpha^c\left(\rho_+^c\right)\right)} + \left(r+\delta^d\right)\frac{\kappa^d}{\overline{m}^d}\left(\alpha_p^d\right)^{1-\varepsilon} = \rho_p^c - \rho_p^d \tag{43}$$

Appendix E.3 shows how to obtain the equilibrium conditions for market tightness.

The next proposition shows the efficiency of the competitive equilibrium with this mechanism to set interest rates.

Proposition 3 *Interest rate posting makes the competitive financial intermediation efficient.*

Proof. *The competitive economy is characterized by* $\left\{\alpha_p^x, \rho_p^x\right\}_{x=c,d}$ *with* $\alpha_p^x = \alpha_o^x$ *for* $x = c, d$. *See Appendix E.4.* ■

Financial intermediation is constraint efficient when non-financial agents direct their search effort and banks posted interest rates. If banks post interest rates on markets instead of bargaining over them with non-financial agents, the potential market failure identified above disappears. Banks jointly determine the search efforts and the interest rates and are not exposed to the double loss in the case of bargaining failure as described in Section 4.

5.4. Interbank Market Frictions

This Section introduces matching frictions on the interbank market in line with Duffie et al. (2005), Lagos and Rocheteau (2009), and Lagos et al. (2011). Banks are now specialized either in the deposit activity or in the credit activity. Deposit banks search depositors on the deposit market but cannot transform these deposits into final good consumption. Credit banks search borrowers on the credit market, which are able to transform deposits into final good consumption.

The populations of deposit and credit bank are both equal to the unity. At each period, there are $0 < \sigma \leq 1$ old matches of credit and deposit banks that perform financial intermediation and $(1 - \sigma)$ new created matches. In new matches, a contract is written between the two banks which stipulates the quantity of depositors $\widetilde{n}$ brought by the deposit bank to finance the $\widetilde{n}$ borrowers selected by the credit bank. Because credit banks receive interests from borrowers while deposit banks pay interests to depositor, the contract stipulates a payment $\widetilde{\varrho}$ from the credit bank to the deposit bank. Thereafter, each bank determines independently the optimal strategy to recruit the $\widetilde{n}$ customers. With a probability σ an existing match between specialized banks is dissolved, which leads to the breakdown of relationships with non-financial customers. The parameter σ measures the size of frictions on the interbank market.

5.4.1. Financial Contract

Specialized banks decide on the recruitment strategy defined by $\left\{\varrho_+^x, v^x, \rho_+^x\right\}$ but not on the masses of non-financial customers n^x, for $x = \{c, d\}$, that should be equal to $\widetilde{n}$. Indeed, the masses of non-financial customers should be decided jointly to avoid waste in search activities (depositors or borrowers would be uselessly recruited). This decision is taken simultaneously with the interbank payment $\widetilde{\varrho}$. The Nash solution for this bargaining is

$$\{\widetilde{n}, \widetilde{\varrho}\} = \arg\max \left\{ \left(P^d(0,0) + \widetilde{\varrho}\right)^{\widetilde{\eta}} \left(P^c(0,0) - \widetilde{\varrho}\right)^{1-\widetilde{\eta}} \right\} \tag{44}$$

where $P^x(\widetilde{n}, \varrho^x)$ is the value function of the bank specialized in market x given the pre-determined values of $\widetilde{n}$ and ϱ^x, for $x = \{c, d\}$. When specialized banks meet and bargain, both banks have no customers and therefore do not pay or receive interests, hence $\widetilde{n} = \varrho^x = 0$. If bargaining succeeds, specialized banks get the value function $P^x(0,0)$ more or less the payment $\widetilde{\varrho}$. Otherwise, if bargaining breaks down, specialized banks get zero payoff.

The value function of the deposit bank is

$$\begin{aligned} P^d\left(\widetilde{n}, \varrho^d\right) = &\max_{\varrho^d_+, v^d, \rho^d_+} \left\{ -\varrho^d - \kappa^d v^d + \beta\left[\sigma P^d\left(\widetilde{n}, \varrho^d_+\right) + (1-\sigma) P^d(0,0)\right] \right\} \\ &-\lambda^d\left[\widetilde{n} - \left(1-\delta^d\right)\widetilde{n} - q\left(\alpha^d\left(\rho^d_+\right)\right) v^d\right] - \mu^d\left[\varrho^d_+ - \left(1-\delta^d\right)\varrho^d - q\left(\alpha^d\left(\rho^d_+\right)\right) v^d \rho^d_+\right] \end{aligned} \tag{45}$$

With a probability $(1-\sigma)$ the match is dissolved and the deposit bank obtains the value function of the bank in a newly created matched, that is $P^d(0,0)$. Similarly, the value function of the credit bank is denoted $P^c(\widetilde{n}, \varrho^c)$ and satisfies

$$\begin{aligned} P^c\left(\widetilde{n}, \varrho^c\right) = &\max_{\varrho^c_+, v^c, \rho^c_+} \left\{ \varrho^c - \kappa^c v^c + \beta\left[\sigma P^c\left(\widetilde{n}, \varrho^c_+\right) + (1-\sigma) P^c(0,0)\right] \right\} \\ &-\lambda^c\left[\widetilde{n} - \left(1-\delta^c\right)\widetilde{n} - q\left(\alpha^c\left(\rho^c_+\right)\right) v^c\right] - \mu^c\left[\varrho^c_+ - \left(1-\delta^c\right)\varrho^c - q\left(\alpha^c\left(\rho^c_+\right)\right) v^c \rho^c_+\right] \end{aligned} \tag{46}$$

With a probability $(1-\sigma)$ the match is dissolved and the credit bank obtaines the value function of the bank in a newly created matched, that is $P^c(0,0)$.

The interbank payment solution is

$$(1+r^*)(1-\beta)\,\widetilde{\varrho} = \widetilde{\eta}\left[\varrho^c - r^*\kappa^c v^c_0 - \kappa^c v^c\right] + (1-\widetilde{\eta})\left[\varrho^d + r^*\kappa^d v^d_0 + \kappa^d v^d\right] \tag{47}$$

where $(1+r^*) = (\beta\sigma)^{-1} \geq (1+r)$ is the discount rate given the probability σ of separation. The resolution of the financial contract (44) is detailled in Section F. The interbank payment is the average of the discounted bank's profits weighted by the bargaining powers. For $\widetilde{\eta} = 0$, the deposit bank has no bargaining power and the payment covers the costs of financial services:

the deposit interests $\left(\varrho^d\right)$ plus the search costs per period $\left(\kappa^d v^d\right)$ and the discounted search costs at the first period $\left(r^* \kappa^d v_0^d\right)$. For $\widetilde{\eta} = 1$, the credit bank has no bargaining power and the payment equals its profits: the credit interests (ϱ^c) less the search costs per period $(\kappa^c v^c)$ and the discounted search costs at the first period $(r^* \kappa^c v_0^c)$. All other things being equal, the interbank market payment increases with the deposit and credit interest rates and the search costs on the deposit market while it decreases with the search costs on the credit market.

5.4.2. Equilibrium

Definition 3 *The competitive financial intermediation with posted interest rates by specialized banks is characterized by the interest rates $\{\rho_s^x\}_{x=\{c,d\}}$ that satisfy*

$$\rho_s^d = \rho^h + \left(\frac{1-\varepsilon}{\varepsilon}\right)\left(r^* + \delta^d\right)\frac{\kappa^d}{\overline{m}^d}\left(\alpha_s^d\right)^{1-\varepsilon} \tag{48}$$

$$\rho_s^c = z - \left(\frac{1-\varepsilon}{\varepsilon}\right)\left[\left(r^* + \delta^c\right)\frac{\kappa^c}{\overline{m}^c}\left(\alpha_s^c\right)^{1-\varepsilon} + \kappa^c \alpha_s^c\right] \tag{49}$$

where $(1 + r^*) = (\beta\sigma)^{-1} \geq (1 + r)$. The equilibrium market tightness variables $\{\alpha_s^x\}_{x=\{c,d\}}$ are the solution of

$$\alpha_s^d = \frac{\kappa^h}{\kappa^d}\left(\frac{\varepsilon}{1-\varepsilon}\right) \tag{50}$$

$$\left(r^* + \delta^c\right)\frac{\kappa^c}{\overline{m}^c}\left(\alpha_s^c\right)^{1-\varepsilon} + \left(r^* + \delta^d\right)\frac{\kappa^d}{\overline{m}^d}\left(\alpha_s^d\right)^{1-\varepsilon} = \rho_s^c - \rho_s^d \tag{51}$$

Appendix F shows how to obtain the equilibrium conditions for market tightness. The mass of financed entrepreneurs is $\sigma n^c(\alpha^c)$ where the function $n^c(\alpha^c)$ is defined in Section A.

The consequences of interbank market frictions are characterized in the next Proposition.

Proposition 4 *When σ falls below the unity, interbank market frictions make inefficient the competitive financial intermediation: credit rationing occurs and interest rates are too high.*

Proof. *It is straightforward to see that for $\sigma = 1$ the competitive equilibrium defined by Definition 3 coincides with that of Definition 2, since $r^* = r$, which efficiency is demonstrated* *in Proposition 3. To consider the effects of a fall in σ below unity, notice first that the tightness α_s^d is independant on σ according to (50). Then, α_s^c solves (51), which can be expressed as follows*

$$\left(r^*+\delta^c\right)\frac{\kappa^c}{m^c}\left(\alpha_s^c\right)^{1-\varepsilon}+\left(r^*+\delta^d\right)\frac{\kappa^d}{m^d}\left(\frac{\kappa^h}{\kappa^d}\left(\frac{\varepsilon}{1-\varepsilon}\right)\right)^{1-\varepsilon}+\left(1-\varepsilon\right)\kappa^c\alpha_s^c=\varepsilon\left(z-\rho^h\right) \quad (52)$$

A fall in σ makes higher r^* and therefore the LHS of (52). To restore the efficiency of search efforts by banks, α_s^c should decrease to lower matching costs on the credit market. The deposit interest rate ρ_s^d increases because a fall in σ implies an increase in r^* in (48) for the value of α_s^d given by (50). Using (52), the credit interest rate ρ_s^c defined by (49) solves

$$\rho_s^c=z-\left(\frac{1-\varepsilon}{\varepsilon}\right)\left[\varepsilon\left(z-\rho^h\right)+\varepsilon\kappa^c\alpha_s^c-\left(r^*+\delta^d\right)\frac{\kappa^d}{m^d}\left(\frac{\kappa^h}{\kappa^d}\left(\frac{\varepsilon}{1-\varepsilon}\right)\right)^{1-\varepsilon}\right] \quad (53)$$

it increases because a fall in σ implies an increase in r^* and a decrease in α_s^c .The interest margin is deduced from (48) and (49) as

$$\rho^c-\rho^d=\varepsilon\left(z-\rho^h\right)-\left(1-\varepsilon\right)\kappa^c\alpha_s^c \quad (54)$$

it increases because a fall in σ implies a decrease in α_s^c. ■

The model is therefore consistent with episodes of financial crisis where interbank market failure leads to a credit crunch in the real economy with an increase of the interest rates on the retail markets; e.g. Adrian et al. (2012) and Brunnermeier (2009) for the 2007-2008 crisis.

6. Conclusion

I proposed in this paper a new model of financial intermediation based on search frictions. Banks should invest in search activities on the two markets, credit and deposit, to realize their financial intermediation activities. The use of the search-theoretic approach allows the definition of the constrained efficient financial intermediation and the discussion of the condition of its achievement in a competitive environment. I identified a specific source of market failures, which is the consequence of the simultaneous search processes managed by banks. When a bank bargains with a depositor (or a creditor), it considers that if the bargaining process fails, a creditor will no longer be financed (or a deposit will no longer be paid). This context alters the outcome of the bargaining process, and the traditional Hosios (1990) condition for efficiency is no longer sufficient. My contribution to the search literature is therefore to generalize the Hosios (1990) condition to the case of search on two frictional markets.

Efficient bargaining powers are however complex functions of the structural parameters and may be difficult to implement. Posted interest rates by banks and directed search by non-financial agents is another way to reach efficiency. This price strategy, however, requires strong assumptions. The posted interests are not for one contract (the first credit to the entrepreneur, for example), but they are instead for all contracts repeated during the financial relationships. Therefore, it requires a strong commitment of banks to future interest rates. This commitment may be challenged by the individual interests of agents to renegotiate contracts, especially in times of financial crisis. Indeed, crises make it harder to commit because financial contracts are generally not state-contingent on variables such as business failure risk. Several institutions, such as the European Commission (2013) and the U.K. Independent Commission on Banking (2001), strongly recommend reducing search and switching costs in banking retail markets, which are still high, as explained in the Section 2. Besides the welfare losses supported by the consumers and the reduction of the market's size induced by these costs, the results reported in this paper support these recommendations given the strong assumptions required to achieve an efficient financial intermediation with search frictions.

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A. Proof of Lemma 1

The number of matched entrepreneurs solution of (1) is a function of the credit market tightness α^c

$$n^c(\alpha^c) = \frac{\overline{m}^c (\alpha^c)^\varepsilon}{\delta^c + \overline{m}^c (\alpha^c)^\varepsilon} \overline{n}^c \tag{A.1}$$

The ratio $n^c(\alpha^c)/\overline{n}^c$ measures the degree of financial intermediation as the rate of financing entrepreneurs.

The number of searching households solution of (2) is a function of the two tightness variables $\{\alpha^c, \alpha^d\}$

$$u^d\left(\alpha^d, \alpha^c\right) = \frac{\delta^d}{\overline{m}^d (\alpha^d)^\varepsilon} n^c(\alpha^c) \tag{A.2}$$

where the constraint (3) is saturated: $n^d = n^c(\alpha^c)$. Because it is costly to collect deposits, at the equilibrium all deposits are lent. The welfare is a function of the two tightness variables $\{\alpha^c, \alpha^d\}$

$$U\left(\alpha^c, \alpha^d\right) = \frac{1}{1-\beta}\left[\begin{array}{c} n^c(\alpha^c) z + \left(\overline{n}^d - n^c(\alpha^c)\right)\rho^h \\ -\kappa^h u^d\left(\alpha^d, \alpha^c\right) - \kappa^d \alpha^d u^d\left(\alpha^d, \alpha^c\right) - \kappa^c \alpha^c\left(\overline{n}^c - n^c(\alpha^c)\right) \end{array}\right] \tag{A.3}$$

that is the final good production done by financing entrepreneurs, plus the final good production done by households, less search costs paid by searching households and banks.

B. Proof of Proposition 1

The socially optimal allocation is the solution of (5), which first order conditions are

$$v^x : \kappa^x = \lambda^x \frac{\partial m^x(u^x, v^x)}{\partial v^x}, \text{ for } x = \{c, d\} \tag{B.1}$$

$$n_+^x : \lambda^x = \beta \frac{\partial O\left(n_+^c, n_+^d\right)}{\partial n_+^x}, \text{ for } x = \{c, d\} \tag{B.2}$$

$$u^d : \kappa^h = \lambda^d \frac{\partial m^d\left(u^d, v^d\right)}{\partial u^d} \tag{B.3}$$

The contributions to the value function of the marginal credit is

$$\frac{\partial O\left(n^c, n^d\right)}{\partial n^c} = z + \lambda^c \left[(1-\delta^c) - \frac{\partial m^c\left(\overline{n}^c - n^c, v^c\right)}{\partial\left(\overline{n}^c - n^c\right)}\right] - \lambda^i \tag{B.4}$$

and of the marginal deposit is

$$\frac{\partial O\left(n^c, n^d\right)}{\partial n^d} = -\rho^h + \lambda^d\left(1-\delta^d\right) + \lambda^i \tag{B.5}$$

The value of the multiplier λ^d given by (B.1) is introduced in the optimal condition (B.3) to get

$$\kappa^h = \lambda^d \frac{\partial m^d\left(u^d, v^d\right)}{\partial u^d} = \frac{\kappa^d}{\partial m^d\left(u^d, v^d\right)/\partial v^d}\frac{\partial m^d\left(u^d, v^d\right)}{\partial u^d} = \kappa^d \frac{\partial m^d\left(u^d, v^d\right)/\partial u^d}{\partial m^d\left(u^d, v^d\right)/\partial v^d} \tag{B.6}$$

For the matching function (4), it becomes

$$\alpha^d = \frac{\kappa^h}{\kappa^d}\frac{\varepsilon}{1-\varepsilon} \tag{B.7}$$

The equations (B.2) and (B.5) are used to get the value of the multiplier λ^i

$$\lambda^i = \rho^h + \left(r + \delta^d\right)\lambda^d \tag{B.8}$$

Remember that $r = 1/\beta - 1$. Equations (B.2) and (B.4) give

$$\lambda^c = \beta\left\{z + \lambda^c\left[(1-\delta^c) - \frac{\partial m^c\left(\overline{n}^c - n^c, v^c\right)}{\partial\left(\overline{n}^c - n^c\right)}\right] - \lambda^i\right\} \tag{B.9}$$

By using (B.8) to suppress λ^i, it becomes

$$(r+\delta^c)\lambda^c + \left(r+\delta^d\right)\lambda^d = z - \rho^h - \frac{\partial m^c\left(\overline{n}^c - n^c, v^c\right)}{\partial\left(\overline{n}^c - n^c\right)}\lambda^c \tag{B.10}$$

By using (B.1) and (4), it becomes

$$(r+\delta^c)\frac{\kappa^c}{\overline{m}^c}(\alpha^c)^{1-\varepsilon} + \left(r+\delta^d\right)\frac{\kappa^d}{\overline{m}^d}\left(\alpha^d\right)^{1-\varepsilon} = \varepsilon\left(z-\rho^h\right) - (1-\varepsilon)\kappa^c\alpha^c \tag{B.11}$$

The socially optimal market tightness variables $\{\alpha_o^c, \alpha_d^c\}$ solve (B.7) and (B.11). Equation (B.7) gives the unique and strictly positive value for α_o^d in function of the structural parameters $\left\{\kappa^h, \kappa^d, \varepsilon\right\}$, see (6). The value α_o^c solves (B.11) in function of α_o^d and other structural parameters, see also (7). Equation (B.11) is rearranged as follows

$$(r+\delta^c)\frac{\kappa^c}{\overline{m}^c}(\alpha_o^c)^{1-\varepsilon} + (1-\varepsilon)\kappa^c\alpha_o^c = \varepsilon\left(z-\rho^h\right) - \left(r+\delta^d\right)\frac{\kappa^d}{\overline{m}^d}\left(\alpha_o^d\right)^{1-\varepsilon} \tag{B.12}$$

The RHS term is independent of α_o^c, whereas the LHS term is strictly increasing with α_o^c and equal to 0 for $\alpha_o^c = 0$. To ensure a positive value for α_o^c, the LHS term should be strictly positive, that is

$$\left(z - \rho^h\right) > \left(r + \delta^d\right) \frac{\kappa^d}{\varepsilon \overline{m}^d} \left(\alpha_o^d\right)^{1-\varepsilon} \tag{B.13}$$

$\left(z - \rho^h\right)$ is the gap between the household and entrepreneur technologies, $\left(r + \delta^d\right)$ the discounted rate for time preference r and for exogenous separation δ^d, and κ^d, the per-period search cost, is divided by the marginal productivity of banks' search effort on the deposit market, $\varepsilon \overline{m}^d \left(\alpha_o^d\right)^{\varepsilon-1}$. If the condition (B.13) is not satisfied, there is no financial intermediation. It is socially optimal that households use all their raw goods to produce the final good and that entrepeneurs do not produce.

C. Competitive Financial Intermediation with Interest Rate Bargaining

C.1. Bank's Search Efforts

The first order conditions of the program (14) are

$$v^x : \frac{\kappa^x}{q\left(\alpha^x\right)} = \lambda^x, \text{ for } x = \{c, d\} \tag{C.1}$$

$$n_+^c : \lambda^c = \beta \frac{\partial B\left(n_+^c, n_+^d\right)}{\partial n_+^c} = \beta \left[\rho^c + \left(1 - \delta^c\right) \lambda^c - \lambda^i\right] \tag{C.2}$$

$$n_+^d : \lambda^d = \beta \frac{\partial B\left(n_+^c, n_+^d\right)}{\partial n_+^d} = \beta \left[-\rho^d + \left(1 - \delta^d\right) \lambda^d + \lambda^i\right] \tag{C.3}$$

Equations (C.2) and (C.3) give two expressions for λ^i

$$\lambda^i = \rho^c - \left(r + \delta^c\right) \lambda^c \tag{C.4}$$

and

$$\lambda^i = \rho^d + \left(r + \delta^d\right) \lambda^d \tag{C.5}$$

The equality between these two expressions for λ^i implies

$$\left(r+\delta^c\right)\lambda^c+\left(r+\delta^d\right)\lambda^d=\rho^c-\rho^d \tag{C.6}$$

Using the first order conditions (C.1) to get the expressions for λ^x, it becomes (15) for the matching functions (4).

C.2. Bank's Surplus

The bank's surplus associated with the marginal credit and deposit are

$$\begin{aligned}&\left.\frac{\partial B\left(n^c,n^d\right)}{\partial n^c}\right|_{n^d=n^d(n^c)} \\ =\;& \rho^c-\rho^d n_1^d\left(n^c\right)+\lambda^c\left(1-\delta^c\right)+\lambda^d\left(1-\delta^d\right)n_1^d\left(n^c\right)-\lambda^i\left(1-n_1^d\left(n^c\right)\right)\end{aligned} \tag{C.7}$$

and

$$\begin{aligned}&\left.\frac{\partial B\left(n^c,n^d\right)}{\partial n^d}\right|_{n^c=n^c\left(n^d\right)} \\ =\;& \rho^c n_1^c\left(n^d\right)-\rho^d+\lambda^c\left(1-\delta^c\right)n_1^c\left(n^d\right)+\lambda^d\left(1-\delta^d\right)-\lambda^i\left(n_1^c\left(n^d\right)-1\right)\end{aligned} \tag{C.8}$$

Using the expressions for λ_i provided by (C.4) and (C.5), and the first order conditions (C.1), surplus are

$$\begin{aligned}&\left.\frac{\partial B\left(n^c,n^d\right)}{\partial n^c}\right|_{n^d=n^d(n^c)} \\ =\;& \rho^c-\rho^d+\left(1-\delta^c\right)\frac{\kappa^c}{q\left(\alpha^c\right)}+\left(1-\delta^d\right)\frac{\kappa^d}{q\left(\alpha^d\right)}-\left[1-n_1^d\left(n^c\right)\right]\left(1+r\right)\frac{\kappa^d}{q\left(\alpha^d\right)}\end{aligned} \tag{C.9}$$

and

$$\begin{aligned}&\left.\frac{\partial B\left(n^c,n^d\right)}{\partial n^d}\right|_{n^c=n^c\left(n^d\right)} \\ =\;& \rho^c-\rho^d+\left(1-\delta^c\right)\frac{\kappa^c}{q\left(\alpha^c\right)}+\left(1-\delta^d\right)\frac{\kappa^d}{q\left(\alpha^d\right)}-\left[1-n_1^c\left(n^d\right)\right]\left(1+r\right)\frac{\kappa^c}{q\left(\alpha^c\right)}\end{aligned} \tag{C.10}$$

C.3. Nash Bargaining

For the value functions (8)-(9)-(10), and using the optimality condition on the deposit market (11), the household's surplus is

$$
\begin{aligned}
D^m - D^u &= \rho^d - \rho^h + \left(1-\delta^d\right)\beta\left[D^m\left(\rho^d\right) - D^u\right] \\
&= \rho^d - \rho^h + \left(1-\delta^d\right)\frac{\kappa^h}{p\left(\alpha^d\right)}
\end{aligned}
\tag{C.11}
$$

For the value functions (12)-(13), and using the first order condition of the Nash solution (16), the entrepreneur's surplus is

$$
\begin{aligned}
C^m - C^u &= z - \rho^c + \left(1-\delta^c - p\left(\alpha^c\right)\right)\beta\left(C^m - C^u\right) \\
&= z - \rho^c + \left(1-\delta^c - p\left(\alpha^c\right)\right)\beta\frac{\eta^c}{1-\eta^c}\Delta B^c
\end{aligned}
\tag{C.12}
$$

The surplus (C.10) and (C.11) are introduced in the first order condition of the Nash solution (16) to get

$$
\begin{aligned}
&\rho^d - \rho^h + \left(1-\delta^d\right)\frac{\kappa^h}{p\left(\alpha^d\right)} \\
= &\frac{\eta^d}{1-\eta^d}\left\{\rho^c - \rho^d + \left(1-\delta^c\right)\frac{\kappa^c}{q\left(\alpha^c\right)} + \left(1-\delta^d\right)\frac{\kappa^d}{q\left(\alpha^d\right)} - \left[1 - n_1^c\left(n^d\right)\right]\left(1+r\right)\frac{\kappa^c}{q\left(\alpha^c\right)}\right\}
\end{aligned}
\tag{C.13}
$$

Using the optimality condition (15), it becomes

$$
\begin{aligned}
&\rho^d - \rho^h + \left(1-\delta^d\right)\frac{\kappa^h}{p\left(\alpha^d\right)} \\
= &\frac{\eta^d}{1-\eta^d}\left\{\left(1+r\right)\frac{\kappa^c}{q\left(\alpha^c\right)} + \left(1+r\right)\frac{\kappa^d}{q\left(\alpha^d\right)} - \left[1 - n_1^c\left(n^d\right)\right]\left(1+r\right)\frac{\kappa^c}{q\left(\alpha^c\right)}\right\}
\end{aligned}
\tag{C.14}
$$

and

$$
\rho^d = \rho^h - \left(1-\delta^d\right)\frac{\kappa^h}{p\left(\alpha^d\right)} + \left(1+r\right)\left(\frac{\eta^d}{1-\eta^d}\right)\left[\frac{\kappa^d}{q\left(\alpha^d\right)} + n_1^c\left(n^d\right)\frac{\kappa^c}{q\left(\alpha^c\right)}\right]
\tag{C.15}
$$

The surplus (C.12) is introduced into the first order condition of the Nash solution (16) to get

$$
\rho^c = z - \left(r + \delta^c + p\left(\alpha^c\right)\right)\frac{\eta^c}{1-\eta^c}\beta\Delta B^c
\tag{C.16}
$$

For the bank's surplus (C.9), it becomes

$$\begin{aligned}\rho^c &= z-(r+\delta^c+p(\alpha^c))\frac{\eta^c}{1-\eta^c}\beta\left[\rho^c-\rho^d+(1-\delta^c)\frac{\kappa^c}{q(\alpha^c)}+\left(1-\delta^d\right)\frac{\kappa^d}{q(\alpha^d)}\right] \\ &\quad +(r+\delta^c+p(\alpha^c))\frac{\eta^c}{1-\eta^c}\left[1-n_1^d(n^c)\right]\frac{\kappa^d}{q(\alpha^d)}\end{aligned} \tag{C.17}$$

Using the optimality condition (15), it becomes

$$\rho^c = z-(r+\delta^c+p(\alpha^c))\left(\frac{\eta^c}{1-\eta^c}\right)\left[\frac{\kappa^c}{q(\alpha^c)}+n_1^d(n^c)\frac{\kappa^d}{q(\alpha^d)}\right] \tag{C.18}$$

or, equivalently

$$\rho^c = z-[1+r+p(\alpha^c)-(1-\delta^c)]\left(\frac{\eta^c}{1-\eta^c}\right)\left[\frac{\kappa^c}{q(\alpha^c)}+n_1^d(n^c)\frac{\kappa^d}{q(\alpha^d)}\right] \tag{C.19}$$

C.4. Equilibrium

The competitive market tightness variables are $\{\alpha_b^x,\rho_b^x\}_{x=c,d}$. Introducing the expression of ρ_b^d given by (C.15) into (11) gives

$$\frac{\kappa^h}{p\left(\alpha_b^d\right)} = \left(\frac{\eta^d}{1-\eta^d}\right)\left[\frac{\kappa^d}{q\left(\alpha_b^d\right)}+n_1^c\left(n_b^d\right)\frac{\kappa^c}{q\left(\alpha_b^c\right)}\right] \tag{C.20}$$

which corresponds to (22) for the specification (4) of the matching functions. For the expressions of ρ_b^d given by (C.15) and of ρ_b^c given by (C.19), the bank's net interest margin is

$$\begin{aligned}\rho_b^c-\rho_b^d &= z-\rho^h-(r+\delta^c+p(\alpha_b^c))\left(\frac{\eta^c}{1-\eta^c}\right)\left[\frac{\kappa^c}{q\left(\alpha_b^c\right)}+n_1^d\left(n_b^c\right)\frac{\kappa^d}{q\left(\alpha_b^d\right)}\right] \\ &\quad +\left(1-\delta^d\right)\frac{\kappa^h}{p\left(\alpha_b^d\right)}-(1+r)\left(\frac{\eta^d}{1-\eta^d}\right)\left[\frac{\kappa^d}{q\left(\alpha_b^d\right)}+n_1^c\left(n_b^d\right)\frac{\kappa^c}{q\left(\alpha_b^c\right)}\right]\end{aligned} \tag{C.21}$$

Using (C.20), (C.21) becomes

$$\begin{aligned}\rho_b^c-\rho_b^d &= z-\rho^h-(r+\delta^c+p(\alpha_b^c))\left(\frac{\eta^c}{1-\eta^c}\right)\left[\frac{\kappa^c}{q\left(\alpha_b^c\right)}+n_1^d\left(n_b^c\right)\frac{\kappa^d}{q\left(\alpha_b^d\right)}\right] \\ &\quad -\left(r+\delta^d\right)\left(\frac{\eta^d}{1-\eta^d}\right)\left[\frac{\kappa^d}{q\left(\alpha_b^d\right)}+n_1^c\left(n_b^d\right)\frac{\kappa^c}{q\left(\alpha_b^c\right)}\right]\end{aligned} \tag{C.22}$$

The expression of the net interest margin given by (C.22) is therefore introduced into (15) to get

$$\begin{aligned}
& \left(r+\delta^{c}\right) \frac{\kappa^{c}}{q\left(\alpha_{b}^{c}\right)}+\left(r+\delta^{d}\right)\left(\frac{1-\eta^{c}}{1-\eta^{d}}\right) \frac{\kappa^{d}}{q\left(\alpha_{b}^{d}\right)} \\
= & \left(1-\eta^{c}\right)\left(z-\rho^{h}\right)-\eta^{c} \kappa^{c} \alpha_{b}^{c}-\left(r+\delta^{c}+p\left(\alpha_{b}^{c}\right)\right) \eta^{c} n_{1}^{d}\left(n_{b}^{c}\right) \frac{\kappa^{d}}{q\left(\alpha_{b}^{d}\right)} \\
& -\left(r+\delta^{d}\right)\left(1-\eta^{c}\right)\left(\frac{\eta^{d}}{1-\eta^{d}}\right) n_{1}^{c}\left(n_{b}^{d}\right) \frac{\kappa^{c}}{q\left(\alpha_{b}^{c}\right)}
\end{aligned} \tag{C.23}$$

which corresponds to (23) for the specification (4) of the matching functions.

C.5. Proof of Proposition 2

The competitive equilibrium is constrained-efficient for specific values for the bargaining powers. To obtain these values, the optimality condition (C.20) is expressed as follows

$$\eta^{d}=\frac{\kappa^{h}}{\alpha_{b}^{d} \kappa^{d}}\left[1+n_{1}^{c}\left(n_{b}^{d}\right) \frac{\kappa^{c}}{\kappa^{d}} \frac{q\left(\alpha_{b}^{d}\right)}{q\left(\alpha_{b}^{c}\right)}+\frac{\kappa^{h}}{\alpha_{b}^{d} \kappa^{d}}\right]^{-1} \tag{C.24}$$

Assuming $\alpha_b^x = \alpha_o^x$ and using the value for α_o^d given by (6), it gives (24). Then, the optimality condition (C.23) is expressed as follows

$$\begin{aligned}
\left(r+\delta^{c}\right) \frac{\kappa^{c}}{q\left(\alpha_{b}^{c}\right)}+\left(r+\delta^{d}\right) \frac{\kappa^{d}}{q\left(\alpha_{b}^{d}\right)} = & \left(1-\eta^{c}\right)\left(z-\rho^{h}\right) \\
& -\eta^{c} \kappa^{c} \alpha_{b}^{c}-\left(r+\delta^{d}\right)\left(\frac{\eta^{d}-\eta^{c}}{1-\eta^{d}}\right) \frac{\kappa^{d}}{q\left(\alpha_{b}^{d}\right)} \\
& -\left(r+\delta^{c}+p\left(\alpha_{b}^{c}\right)\right) \eta^{c} n_{1}^{d}\left(n_{b}^{c}\right) \frac{\kappa^{d}}{q\left(\alpha_{b}^{d}\right)} \\
& -\left(r+\delta^{d}\right)\left(1-\eta^{c}\right)\left(\frac{\eta^{d}}{1-\eta^{d}}\right) n_{1}^{c}\left(n_{b}^{d}\right) \frac{\kappa^{c}}{q\left(\alpha_{b}^{c}\right)}
\end{aligned} \tag{C.25}$$

where the LHS term is identical to that of (7) for $\alpha_b^x = \alpha_o^x$. Because (C.25) is linear with respect to η^c, it is possible to rearrange the terms to get the expression of η_o^c such that the

RHS term of (C.25) is equal to the RHS term of (7). It is

$$\eta_o^c = (1-\varepsilon) \times \frac{z - \rho^h + \kappa^c \alpha_o^c - \left(\frac{r+\delta^d}{1-\eta_o^d}\right)\left(\frac{\eta_o^d}{1-\varepsilon}\right)\left[\frac{\kappa^d}{q\left(\alpha_o^d\right)} + n_1^c\left(n_b^d\right)\frac{\kappa^c}{q(\alpha_o^c)}\right]}{z - \rho^h + \kappa^c \alpha_o^c - \left(\frac{r+\delta^d}{1-\eta_o^d}\right)\left[\frac{\kappa^d}{q\left(\alpha_o^d\right)} + \eta_o^d n_1^c\left(n_b^d\right)\frac{\kappa^c}{q(\alpha_o^c)}\right] + \left(r+\delta^c+p\left(\alpha_o^c\right)\right) n_1^d\left(n_b^c\right)\frac{\kappa^d}{q\left(\alpha_o^d\right)}} \tag{C.26}$$

I then introduce the expression of η_o^d given by (24) into (C.26) to get

$$\eta_o^c = (1-\varepsilon) \times \frac{z - \rho^h + \kappa^c \alpha_o^c - \left(\frac{r+\delta^d}{1-\eta_o^d}\right)\left[\frac{\frac{\kappa^d}{q\left(\alpha_o^d\right)} + n_1^c\left(n_b^d\right)\frac{\kappa^c}{q(\alpha_o^c)}}{1+\varepsilon n_1^c\left(n_b^d\right)\frac{\kappa^c}{\kappa^d}\frac{q\left(\alpha_b^d\right)}{q\left(\alpha_b^c\right)}}\right]}{z - \rho^h + \kappa^c \alpha_o^c - \left(\frac{r+\delta^d}{1-\eta_o^d}\right)\left[\frac{\kappa^d}{q\left(\alpha_o^d\right)} + \frac{(1-\varepsilon)n_1^c\left(n_b^d\right)\frac{\kappa^c}{q(\alpha_o^c)}}{1+\varepsilon n_1^c\left(n_b^d\right)\frac{\kappa^c}{\kappa^d}\frac{q\left(\alpha_b^d\right)}{q\left(\alpha_b^c\right)}}\right] + \left(r+\delta^c+p\left(\alpha_o^c\right)\right) n_1^d\left(n_b^c\right)\frac{\kappa^d}{q\left(\alpha_o^d\right)}} \tag{C.27}$$

and after simplification

$$\eta_o^c = (1-\varepsilon) \times \frac{z - \rho^h + \kappa^c \alpha_o^c - \left(\frac{r+\delta^d}{1-\eta_o^d}\right)\left[\frac{\frac{\kappa^d}{q\left(\alpha_o^d\right)} + n_1^c\left(n_b^d\right)\frac{\kappa^c}{q(\alpha_o^c)}}{1+\varepsilon n_1^c\left(n_b^d\right)\frac{\kappa^c}{\kappa^d}\frac{q\left(\alpha_o^d\right)}{q\left(\alpha_o^c\right)}}\right]}{\begin{array}{c} z - \rho^h + \kappa^c \alpha_o^c - \left(\frac{r+\delta^d}{1-\eta_o^d}\right)\left[\frac{\frac{\kappa^d}{q\left(\alpha_o^d\right)} + n_1^c\left(n_b^d\right)\frac{\kappa^c}{q(\alpha_o^c)}}{1+\varepsilon n_1^c\left(n_b^d\right)\frac{\kappa^c}{\kappa^d}\frac{q\left(\alpha_o^d\right)}{q\left(\alpha_o^c\right)}}\right] \\ + \left(r+\delta^c+p\left(\alpha_o^c\right)\right) n_1^d\left(n_b^c\right)\frac{\kappa^d}{q\left(\alpha_o^d\right)} \end{array}} \tag{C.28}$$

and finally the solution (25), using once again the expression of η_o^d given by (24).

C.6. Proof of Corollary 1

The reference situation is $n_1^c\left(n_b^d\right) = n_1^d\left(n_b^c\right) = 0$, banks are not held up. In this case, the equations (24) and (25) reduce to

$$\eta_o^x = 1 - \varepsilon, \text{ for } x = \{c, d\}. \tag{C.29}$$

which is known as the Hosios (1990) condition for efficiency.

Depositors hold up and not borrowers when $n_1^d\left(n_b^c\right)=1$ and $n_1^c\left(n_b^d\right)=0$. The socially optimal values of bargaining power are $\eta_o^d=(1-\varepsilon)$, according to (24), and

$$\eta_o^c=(1-\varepsilon)\frac{z-\rho^h+\kappa^c\alpha_o^c-\left(r+\delta^d\right)\frac{\kappa^d}{\varepsilon q\left(\alpha_o^d\right)}}{z-\rho^h+\kappa^c\alpha_o^c-\left(r+\delta^d\right)\frac{\kappa^d}{\varepsilon q\left(\alpha_o^d\right)}+\left(r+\delta^c+p\left(\alpha_o^c\right)\right)\frac{\kappa^d}{q\left(\alpha_o^d\right)}}<(1-\varepsilon)\quad\text{(C.30)}$$

according to (25). If η^c remains equal to $(1-\varepsilon)$, the deposit market tightness remains at its socially optimal value, $\alpha_b^d=\alpha_o^d$, but it is not the case for the credit market tightness α_b^c that solves (C.25), or equivalently

$$\begin{aligned}\left(r+\delta^c\right)\frac{\kappa^c}{\overline{m}^c}\left(\alpha_b^c\right)^{1-\varepsilon}+(1-\varepsilon)\kappa^c\alpha_b^c &= \varepsilon\left(z-\rho^h\right)-\left(r+\delta^d\right)\frac{\kappa^d}{\overline{m}^d}\left(\alpha_o^d\right)^{1-\varepsilon} \\ &\quad-\left(r+\delta^c+\overline{m}^c\left(\alpha_b^c\right)^{\varepsilon}\right)(1-\varepsilon)\frac{\kappa^d}{\overline{m}^d}\left(\alpha_o^d\right)^{1-\varepsilon}\end{aligned}\quad\text{(C.31)}$$

where α_o^c solves (7), which can be rewritten as follows

$$\left(r+\delta^c\right)\frac{\kappa^c}{\overline{m}^c}\left(\alpha_o^c\right)^{1-\varepsilon}+(1-\varepsilon)\kappa^c\alpha_o^c=\varepsilon\left(z-\rho^h\right)-\left(r+\delta^d\right)\frac{\kappa^d}{\overline{m}^d}\left(\alpha_o^d\right)^{1-\varepsilon}\quad\text{(C.32)}$$

The LHS terms of (C.31) and (C.32) are identical and increasing with the credit market tightness. Because, the RHS term of (C.31) is strictly lower than the RHS term of (C.32), the competitive credit market tightness is below its optimal value, $\alpha_b^c<\alpha_o^c$, and the rate of financing entrepreneurs is too low. The competitive equilibrium is still unique, if it exists, because the LHS term of (C.31) is increasing with α_b^c whereas the RHS term of (C.31) is decreasing (and the deposit market tightness α_o^d is constant).

Borrowers hold up and not depositors $n_1^d\left(n_b^c\right)=0$ and $n_1^c\left(n_b^d\right)=1$. The socially optimal values of bargaining power are $\eta_o^c=(1-\varepsilon)$, according to (25), and

$$\eta_o^d=(1-\varepsilon)\left[1+\varepsilon\frac{\kappa^c}{\kappa^d}\frac{\overline{m}^d\left(\alpha_o^d\right)^{1-\varepsilon}}{\overline{m}^c\left(\alpha_o^c\right)^{1-\varepsilon}}\right]^{-1}<(1-\varepsilon)\quad\text{(C.33)}$$

according to (24). If η^d remains equal to $(1-\varepsilon)$, the optimality condition (22) becomes

$$\frac{\kappa^h}{\overline{m}^d\left(\alpha_b^d\right)^{\varepsilon}}=\left(\frac{1-\varepsilon}{\varepsilon}\right)\left[\frac{\kappa^d}{\overline{m}^d}\left(\alpha_b^d\right)^{1-\varepsilon}+\frac{\kappa^c}{\overline{m}^c}\left(\alpha_b^c\right)^{1-\varepsilon}\right]\quad\text{(C.34)}$$

for the specification (4). Given the constraint $(\kappa^c/\overline{m}^c)(\alpha_b^c)^{1-\varepsilon} > 0$, the optimality condition (C.34) implies an upper-limit on α_b^d, which is the socially optimal value α_o^d

$$\alpha_b^d < \frac{\kappa^h}{\kappa^d}\frac{\varepsilon}{1-\varepsilon} = \alpha_o^d \tag{C.35}$$

Using (6) to get $\kappa^h = \alpha_o^d \kappa^d (1-\varepsilon)/\varepsilon$, (C.34) becomes

$$\frac{\kappa^d}{\overline{m}^d}\left(\alpha_b^d\right)^{1-\varepsilon}\frac{\alpha_o^d}{\alpha_b^d} = \frac{\kappa^d}{\overline{m}^d}\left(\alpha_b^d\right)^{1-\varepsilon} + \frac{\kappa^c}{\overline{m}^c}\left(\alpha_b^c\right)^{1-\varepsilon} \tag{C.36}$$

The relative gap between optimal and competitive tightness variables is deduced for (C.36) as

$$\frac{\alpha_o^d - \alpha_b^d}{\alpha_b^d} = \frac{(\kappa^c/\overline{m}^c)(\alpha_b^c)^{1-\varepsilon}}{\left(\kappa^d/\overline{m}^d\right)\left(\alpha_b^d\right)^{1-\varepsilon}} > 0 \tag{C.37}$$

For $\eta^c = \eta^d = (1-\varepsilon)$, the optimality condition (23) becomes

$$\begin{aligned} & (r+\delta^c)\frac{\kappa^c}{\overline{m}^c}(\alpha_b^c)^{1-\varepsilon} + (1-\varepsilon)\kappa^c\alpha_b^c + \left(r+\delta^d\right)(1-\varepsilon)\frac{\kappa^c}{\overline{m}^c}(\alpha_b^c)^{1-\varepsilon} \\ = \;& \varepsilon\left(z-\rho^h\right) - \left(r+\delta^d\right)\frac{\kappa^d}{\overline{m}^d}\left(\alpha_b^d\right)^{1-\varepsilon} \end{aligned} \tag{C.38}$$

Using (C.32), (C.38) is rearranged as

$$\begin{aligned} & (r+\delta^c)\frac{\kappa^c}{\overline{m}^c}\left[(\alpha_b^c)^{1-\varepsilon} - (\alpha_o^c)^{1-\varepsilon}\right] + (1-\varepsilon)\kappa^c(\alpha_b^c - \alpha_o^c) \\ = \;& \left(r+\delta^d\right)\frac{\kappa^d}{\overline{m}^d}\left[\left(\alpha_o^d\right)^{1-\varepsilon} - \left(\alpha_b^d\right)^{1-\varepsilon}\right] - \left(r+\delta^d\right)(1-\varepsilon)\frac{\kappa^c}{q\left(\alpha_b^c\right)} \end{aligned} \tag{C.39}$$

Excessive credit rationing occurs if $\alpha_b^c < \alpha_o^c$. Because the RHS term of (C.39) is strictly increasing with the ratio α_b^c/α_o^c and equal to zero for $\alpha_b^c = \alpha_o^c$, the case $\alpha_b^c < \alpha_o^c$ corresponds to a negative value for the LHS term of (C.39), that is

$$\frac{\kappa^d}{\overline{m}^d}\left[\left(\alpha_o^d\right)^{1-\varepsilon} - \left(\alpha_b^d\right)^{1-\varepsilon}\right] < (1-\varepsilon)\frac{\kappa^c}{q\left(\alpha_b^c\right)} \tag{C.40}$$

Excessive credit rationing occurs for a small gap between optimal and competitive tightness variables for the deposit market.

If $n_1^d\left(n_b^c\right) = 1$ and $n_1^c\left(n_b^d\right) = 1$, efficiency requires $\eta_o^x < \varepsilon$ for $x = \{c, d\}$. For $\eta^c = \eta^d = (1-\varepsilon)$, the optimality condition (22) becomes (C.34) and the relation (C.37) holds, with $\alpha_o^d > \alpha_b^d$. The optimality condition (23) becomes

$$\begin{aligned}&\left(r+\delta^c\right)\frac{\kappa^c}{\overline{m}^c}\left(\alpha_b^c\right)^{1-\varepsilon}+\left(1-\varepsilon\right)\kappa^c\alpha_b^c+\left(r+\delta^d\right)\left(1-\varepsilon\right)\frac{\kappa^c}{\overline{m}^c}\left(\alpha_b^c\right)^{1-\varepsilon} \\ = \;&\varepsilon\left(z-\rho^h\right)-\left(r+\delta^d\right)\frac{\kappa^d}{\overline{m}^d}\left(\alpha_b^d\right)^{1-\varepsilon}-\left(r+\delta^c+\overline{m}^c\left(\alpha_b^c\right)^{\varepsilon}\right)\left(1-\varepsilon\right)\frac{\kappa^d}{\overline{m}^d}\left(\alpha_b^d\right)^{1-\varepsilon}\end{aligned} \tag{C.41}$$

Using (C.32), (C.41) is rearranged as

$$\begin{aligned}&\left(r+\delta^c\right)\frac{\kappa^c}{\overline{m}^c}\left[\left(\alpha_b^c\right)^{1-\varepsilon}-\left(\alpha_o^c\right)^{1-\varepsilon}\right]+\left(1-\varepsilon\right)\kappa^c\left(\alpha_b^c-\alpha_o^c\right) \\ = \;&\left(r+\delta^d\right)\frac{\kappa^d}{\overline{m}^d}\left[\left(\alpha_o^d\right)^{1-\varepsilon}-\left(\alpha_b^d\right)^{1-\varepsilon}\right]-\left(r+\delta^d\right)\left(1-\varepsilon\right)\frac{\kappa^c}{\overline{m}^c}\left(\alpha_b^c\right)^{1-\varepsilon} \\ &-\left(r+\delta^c+\overline{m}^c\left(\alpha_b^c\right)^{\varepsilon}\right)\left(1-\varepsilon\right)\frac{\kappa^d}{\overline{m}^d}\left(\alpha_b^d\right)^{1-\varepsilon}\end{aligned} \tag{C.42}$$

Excessive credit rationing occurs if $\alpha_b^c < \alpha_o^c$. Because the RHS term of (C.42) is strictly increasing with the ratio α_b^c/α_o^c and equal to zero for $\alpha_b^c = \alpha_o^c$, the case $\alpha_b^c < \alpha_o^c$ corresponds to a negative value for the LHS term of (C.42), that is

$$\frac{\kappa^d}{\overline{m}^d}\left[\left(\alpha_o^d\right)^{1-\varepsilon}-\left(\alpha_b^d\right)^{1-\varepsilon}\right]<\left(1-\varepsilon\right)\left[\frac{\kappa^c}{\overline{m}^c}\left(\alpha_b^c\right)^{1-\varepsilon}+\frac{r+\delta^c+\overline{m}^c\left(\alpha_b^c\right)^{\varepsilon}}{r+\delta^d}\frac{\kappa^d}{\overline{m}^d}\left(\alpha_b^d\right)^{1-\varepsilon}\right]$$

Excessive credit rationing occurs for a small gap between optimal and competitive tightness variables for the deposit market.

D. Multilateral Bargaining

The surplus are given by (C.11) and (C.12) for the non-financial agents and by

$$\Delta B = \left.\Delta B^c\right|_{n_1^d(n^c)=1} = \left.\Delta B^d\right|_{n_1^c\left(n^d\right)=1} \tag{D.43}$$

for the bank, see (19). The equilibrium values of tightness variables are denoted α_m^x where m stands for multilateral for $x = \{c, d\}$.

The FOC of (27) with respect to ρ^d is: $\eta^d\Delta B = \eta^b\left(D^m - D^u\right)$. Introducing the definition of the value functions, it becomes

$$\eta^d\left[\rho^c-\rho^d+\left(1-\delta^c\right)\frac{\kappa^c}{q\left(\alpha_m^c\right)}+\left(1-\delta^d\right)\frac{\kappa^d}{q\left(\alpha_m^d\right)}\right]=\eta^b\left[\rho^d-\rho^h+\left(1-\delta^d\right)\frac{\kappa^h}{p\left(\alpha_m^d\right)}\right] \tag{D.44}$$

Using the optimal search efforts by banks (15), the bank's surplus is

$$\Delta B = (1+r)\frac{\kappa^c}{q(\alpha_m^c)} + (1+r)\frac{\kappa^d}{q(\alpha_m^d)} \tag{D.45}$$

Then, the deposit interest rate, which satisfies (D.44), is

$$\rho^d = \rho^h - \left(1-\delta^d\right)\frac{\kappa^h}{p(\alpha_m^d)} + \frac{\eta^d}{\eta^b}(1+r)\left[\frac{\kappa^c}{q(\alpha_m^c)} + \frac{\kappa^d}{q(\alpha_m^d)}\right] \tag{D.46}$$

The FOC of (27) with respect to ρ^c is : $\eta^c \Delta B = \eta^b (C^m - C^u)$. Introducing the expressions (C.12) and (D.45), it gives the following expression for the credit interest rate

$$\rho^c = z - [r + \delta^c + p(\alpha_m^c)]\frac{\eta^c}{\eta^b}\left[\frac{\kappa^c}{q(\alpha_m^c)} + \frac{\kappa^d}{q(\alpha_m^d)}\right] \tag{D.47}$$

Introducing the expression of the deposit interest rate (D.46) into the free entry condition (11) gives

$$\frac{\kappa^h}{p(\alpha_m^d)} = \frac{\eta^d}{\eta^b}\left[\frac{\kappa^c}{q(\alpha_m^c)} + \frac{\kappa^d}{q(\alpha_m^d)}\right] \tag{D.48}$$

For the expression of the interest rates (D.46) and (D.47), the net interest margin is

$$\begin{aligned} \rho^c - \rho^d &= z - \rho^h - [r + \delta^c + p(\alpha_m^c)]\frac{\eta^c}{\eta^b}\left[\frac{\kappa^c}{q(\alpha_m^c)} + \frac{\kappa^d}{q(\alpha_m^d)}\right] \\ &\quad - \left(r+\delta^d\right)\frac{\eta^d}{\eta^b}\left[\frac{\kappa^c}{q(\alpha_m^c)} + \frac{\kappa^d}{q(\alpha_m^d)}\right] \end{aligned} \tag{D.49}$$

using (D.48). Introducing (D.49) into (15) gives

$$\begin{aligned} & (r+\delta^c)\frac{\kappa^c}{q(\alpha_m^c)} + \left(r+\delta^d\right)\left(\frac{\eta^b}{\eta^c+\eta^b}\right)\frac{\kappa^d}{q(\alpha_m^d)} \\ = & \left(\frac{\eta^b}{\eta^c+\eta^b}\right)\left(z-\rho^h\right) - \kappa^c\left(\frac{\eta^c}{\eta^c+\eta^b}\right)\frac{p(\alpha_m^c)}{q(\alpha_m^c)} - [(r+\delta^c) + p(\alpha^c)]\left(\frac{\eta^c}{\eta^c+\eta^b}\right)\frac{\kappa^d}{q(\alpha_m^d)} \\ & - \left(r+\delta^d\right)\left(\frac{\eta^d}{\eta^c+\eta^b}\right)\left[\frac{\kappa^c}{q(\alpha_m^c)} + \frac{\kappa^d}{q(\alpha_m^d)}\right] \end{aligned} \tag{D.50}$$

Assuming $\alpha_m^x = \alpha_o^x$ for $x = \{c, d\}$ and given $\eta^b = 1 - \eta^d - \eta^c$, the free entry condition for households (D.48) can be expressed as follows

$$\eta^d = (1-\eta^c)\left(1 + \left(\frac{\varepsilon}{1-\varepsilon}\right)\left[1 + \frac{\kappa^c}{\kappa^d}\frac{q(\alpha_o^d)}{q(\alpha_o^c)}\right]\right)^{-1} \tag{D.51}$$

using (6), and the optimality condition on search effort by banks (D.50) becomes

$$\begin{aligned} & \varepsilon \left(z-\rho^h\right)-(1-\varepsilon)\,\kappa^c\alpha_o^c \\ = \; & \left(\frac{1-\eta^d-\eta^c}{1-\eta^d}\right)\left(z-\rho^h\right)-\kappa^c\left(\frac{\eta^c}{1-\eta^d}\right)\alpha_o^c-\left(r+\delta^d\right)\left(\frac{\eta^d}{1-\eta^d}\right)\frac{\kappa^c}{q\left(\alpha_o^c\right)} \\ & -\left[r+\delta^c+p\left(\alpha_o^c\right)\right]\left(\frac{\eta^c}{1-\eta^d}\right)\frac{\kappa^d}{q\left(\alpha_o^d\right)}-\left(r+\delta^d\right)\left(\frac{\eta^d-\eta^c}{1-\eta^d}\right)\frac{\kappa^d}{q\left(\alpha_o^d\right)} \end{aligned} \tag{D.52}$$

using (7), where α_o^x are independant on η^x for $x=\{c,d\}$. Therefore efficient bargaining powers are $\left\{\eta^c,\eta^d\right\}$ such that conditions (D.51) and (D.52) are satisfied. If bargaining powers of non-financial agents are equal: $\eta^d=\eta^c=\eta$, the value η is then given by (D.51) as

$$\eta=\left(2+\left(\frac{\varepsilon}{1-\varepsilon}\right)\left[1+\frac{\kappa^c}{\kappa^d}\frac{q\left(\alpha_o^d\right)}{q\left(\alpha_o^c\right)}\right]\right)^{-1} \tag{D.53}$$

For this value of η, there is no condition to ensure that (D.52) is satisfied. Indeed (D.52) becomes

$$\begin{aligned} & \varepsilon \left(z-\rho^h\right)-(1-\varepsilon)\,\kappa^c\alpha_o^c \\ = \; & \left(\frac{1-2\eta}{1-\eta}\right)\left(z-\rho^h\right)-\kappa^c\left(\frac{\eta}{1-\eta}\right)\alpha_o^c-\left(r+\delta^d\right)\left(\frac{\eta}{1-\eta}\right)\frac{\kappa^c}{q\left(\alpha_o^c\right)} \\ & -\left[r+\delta^c+p\left(\alpha_o^c\right)\right]\left(\frac{\eta}{1-\eta}\right)\frac{\kappa^d}{q\left(\alpha_o^d\right)} \end{aligned} \tag{D.54}$$

The parameter η should simultaneously satisfy (D.53) and (D.54).

E. Competitive Financial Intermediation with Interest Rate Posting

E.1. Directed Search by Households

The free entry condition on the deposit market implies

$$D^h=D^u \tag{E.1}$$

or equivalently

$$\kappa^h=p\left(\alpha^d\left(\rho_+^d\right)\right)\beta\left[D^m\left(\rho_+^d\right)-D^u\right] \tag{E.2}$$

given (8) and (28). Using (28), (9), and (E.1), the matching surplus is

$$D^m\left(\rho_+^d\right) - D^u = \frac{\rho_+^d - \rho^h}{1-\left(1-\delta^d\right)\beta} \tag{E.3}$$

The free entry condition (E.2) becomes

$$\frac{\kappa^h}{p\left(\alpha^d\left(\rho_+^d\right)\right)} = \frac{\rho_+^d - \rho^h}{r+\delta^d} \tag{E.4}$$

for the surplus (E.3).

The elasticity of the matching probability of banks with respect to posted interest rate is

$$\begin{aligned}\frac{\partial q\left(\alpha^d\left(\rho_+^d\right)\right)}{\partial \rho_+^d}\frac{\rho_+^d}{q\left(\alpha^d\left(\rho_+^d\right)\right)} &= \frac{\partial q\left(\alpha^d\left(\rho_+^d\right)\right)}{\partial p\left(\alpha^d\left(\rho_+^d\right)\right)}\frac{p\left(\alpha^d\left(\rho_+^d\right)\right)}{q\left(\alpha^d\left(\rho_+^d\right)\right)} \\ &\times\frac{\partial p\left(\alpha^d\left(\rho_+^d\right)\right)}{\partial \rho_+^d}\frac{\rho_+^d}{p\left(\alpha^d\left(\rho_+^d\right)\right)}\end{aligned} \tag{E.5}$$

The first term in bracket of the LHS term of (E.5) is determined by the specification of the matching technology. For the specification (4), it is equal to

$$\frac{\partial q\left(\alpha^d\left(\rho_+^d\right)\right)}{\partial p\left(\alpha^d\left(\rho_+^d\right)\right)}\frac{p\left(\alpha^d\left(\rho_+^d\right)\right)}{q\left(\alpha^d\left(\rho_+^d\right)\right)} = \frac{\frac{\partial q\left(\alpha^d\left(\rho_+^d\right)\right)}{\partial \alpha^d}\frac{\alpha^d}{q\left(\alpha^d\left(\rho_+^d\right)\right)}}{\frac{\partial p\left(\alpha^d\left(\rho_+^d\right)\right)}{\partial \alpha^d}\frac{\alpha^d}{p\left(\alpha^d\left(\rho_+^d\right)\right)}} = -\left(\frac{1-\varepsilon}{\varepsilon}\right) < 0 \tag{E.6}$$

The second term in bracket of the LHS term of (E.5) is deduced as follows. First, the matching probability associated with the interest rate ρ_+^d consistent with (30) is

$$p\left(\alpha^d\left(\rho_+^d\right)\right) = p\left(\alpha^d\left(\overline{\rho}_+^d\right)\right)\frac{D^m\left(\overline{\rho}_+^d\right) - D^u}{D^m\left(\rho_+^d\right) - D^u} \tag{E.7}$$

which elasticity is

$$\frac{\partial p\left(\alpha^d\left(\rho_+^d\right)\right)}{\partial \rho_+^d}\frac{\rho_+^d}{p\left(\alpha^d\left(\rho_+^d\right)\right)} = \frac{\rho_+^d}{D^m\left(\rho_+^d\right) - D^u}\left(-\frac{\partial D^m\left(\rho_+^d\right)}{\partial \rho_+^d}\right) \tag{E.8}$$

The partial derivative of the value function associated with the state matched, defined by (9), is

$$\frac{\partial D^m\left(\rho_+^d\right)}{\partial \rho_+^d} = 1 + \left(1-\delta^d\right)\beta\frac{\partial D^m\left(\rho_+^d\right)}{\partial \rho_+^d} = \frac{1}{1-\left(1-\delta^d\right)\beta} \tag{E.9}$$

Elasticity (E.8) becomes with (E.3) and (E.9)

$$\frac{\partial p\left(\alpha^d\left(\rho_+^d\right)\right)}{\partial \rho_+^d}\frac{\rho_+^d}{p\left(\alpha^d\left(\rho_+^d\right)\right)} = -\frac{\rho_+^d}{\rho_+^d-\rho^h} \tag{E.10}$$

Given (E.6) and (E.10), the elasticity (E.5) is therefore equal to

$$\frac{\partial q\left(\alpha_+^d\left(\rho_+^d\right)\right)}{\partial \rho_+^d}\frac{\rho_+^d}{q\left(\alpha^d\left(\rho_+^d\right)\right)} = \left(\frac{1-\varepsilon}{\varepsilon}\right)\left(\frac{\rho_+^d}{\rho_+^d-\rho^h}\right) > 0 \tag{E.11}$$

This last expression can be used to get the following expression of posted deposit interest rate

$$\rho_+^d = \rho^h + \frac{q\left(\alpha^d\left(\rho_+^d\right)\right)}{\partial q\left(\alpha_+^d\left(\rho_+^d\right)\right)/\partial \rho_+^d}\left(\frac{1-\varepsilon}{\varepsilon}\right) \tag{E.12}$$

from (E.5).

E.2. Directed Search by Entrepreneurs

Being matched yields a surplus equal to

$$\begin{aligned} C^m\left(\rho_+^c\right) - C^u &= z-\rho_+^c + \left[1-\delta^c - p\left(\alpha^c\left(\rho_+^c\right)\right)\right]\beta\left[C^m\left(\rho_+^c\right)-C^u\right] \\ &= \frac{z-\rho_+^c}{1-\left[1-\delta^c-p\left(\alpha^c\left(\rho_+^c\right)\right)\right]\beta} \end{aligned} \tag{E.13}$$

The elasticity of the matching probability of banks with respect to posted interest rate is

$$\begin{aligned} \frac{\partial q\left(\alpha^c\left(\rho_+^c\right)\right)}{\partial \rho_+^c}\frac{\rho_+^c}{q\left(\alpha^c\left(\rho_+^c\right)\right)} &= \frac{\partial q\left(\alpha^c\left(\rho_+^c\right)\right)}{\partial p\left(\alpha^c\left(\rho_+^c\right)\right)}\frac{p\left(\alpha^c\left(\rho_+^c\right)\right)}{q\left(\alpha^c\left(\rho_+^c\right)\right)} \\ &\quad\times\frac{\partial p\left(\alpha^c\left(\rho_+^c\right)\right)}{\partial \rho_+^c}\frac{\rho_+^c}{p\left(\alpha^c\left(\rho_+^c\right)\right)} \end{aligned} \tag{E.14}$$

The first term in bracket of the LHS term of (E.14) is determined by the specification of the matching technology. For the specification (4), it is equal to

$$\frac{\partial q\left(\alpha^c\left(\rho_+^c\right)\right)}{\partial p\left(\alpha^c\left(\rho_+^c\right)\right)}\frac{p\left(\alpha^c\left(\rho_+^c\right)\right)}{q\left(\alpha^c\left(\rho_+^c\right)\right)} = \frac{\frac{\partial q\left(\alpha^c\left(\rho_+^c\right)\right)}{\partial\alpha^c}\frac{p\left(\alpha^c\left(\rho_+^c\right)\right)}{\alpha^c}}{\frac{\partial p\left(\alpha^c\left(\rho_+^c\right)\right)}{\partial\alpha^c}\frac{q\left(\alpha^c\left(\rho_+^c\right)\right)}{\alpha^c}} = -\left(\frac{1-\varepsilon}{\varepsilon}\right) \tag{E.15}$$

The second term in bracket of the LHS term of (E.14) is deduced from the no-arbitrage condition (35) as follows

$$\frac{\partial p\left(\alpha^c\left(\rho_+^c\right)\right)}{\partial \rho_+^c}\frac{\rho_+^c}{p\left(\alpha^c\left(\rho_+^c\right)\right)} = \frac{\rho_+^c}{C^m\left(\rho_+^c\right)-C^u}\left(\frac{-\partial C^m\left(\rho_+^c\right)}{\partial \rho_+^c}\right) \tag{E.16}$$

The partial derivative of the value function associated with the matched state, defined by (13), is

$$\frac{\partial C^m\left(\rho_+^c\right)}{\partial \rho_+^c} = -1 + \left(1-\delta^c\right)\beta \frac{\partial C^m\left(\rho_+^c\right)}{\partial \rho_+^c} = \frac{-1}{1-\left(1-\delta^c\right)\beta} \tag{E.17}$$

With (E.13) and (E.17), the elasticity (E.16) becomes

$$\frac{\partial p\left(\alpha^c\left(\rho_+^c\right)\right)}{\partial \rho_+^c}\frac{\rho_+^c}{p\left(\alpha^c\left(\rho_+^c\right)\right)} = -\frac{\rho_+^c}{z-\rho^c}\frac{1-\left[1-\delta^c-p\left(\alpha^c\left(\rho_+^c\right)\right)\right]\beta}{1-\left(1-\delta^c\right)\beta} \tag{E.18}$$

Given (E.15) and (E.18), the elasticity (E.14) is therefore equal to

$$\frac{\partial q\left(\alpha^c\left(\rho_+^c\right)\right)}{\partial \rho_+^c}\frac{\rho_+^c}{q\left(\alpha^c\left(\rho_+^c\right)\right)} = -\left(\frac{1-\varepsilon}{\varepsilon}\right)\frac{1-\left[1-\delta^c-p\left(\alpha^c\left(\rho_+^c\right)\right)\right]\beta}{1-\left(1-\delta^c\right)\beta}\left(\frac{\rho_+^c}{z-\rho_+^c}\right) < 0 \tag{E.19}$$

This last expression can be used to get the following expression of posted credit interest rate

$$\rho_+^c = z + \frac{q\left(\alpha^c\left(\rho_+^c\right)\right)}{\partial q\left(\alpha^c\left(\rho_+^c\right)\right)/\partial \rho_+^c}\left(\frac{1-\varepsilon}{\varepsilon}\right)\frac{1-\left[1-\delta^c-p\left(\alpha^c\left(\rho_+^c\right)\right)\right]\beta}{1-\left(1-\delta^c\right)\beta} \tag{E.20}$$

E.3. Search and Interest Rate Posting by Banks

The first order conditions of the program (38) are

$$n_+^c : \lambda^c = \beta\frac{\partial P\left(n_+^c, n_+^d, \varrho_+^c, \varrho_+^d\right)}{\partial n_+^c} = \beta\left[\left(1-\delta^c\right)\lambda^c - \lambda^i\right] \tag{E.21}$$

$$n_+^d : \lambda^d = \beta\frac{\partial P\left(n_+^c, n_+^d, \varrho_+^c, \varrho_+^d\right)}{\partial n_+^d} = \beta\left[\left(1-\delta^d\right)\lambda^d + \lambda^i\right] \tag{E.22}$$

$$\varrho_+^c : \mu^c = \beta\frac{\partial P\left(n_+^c, n_+^d, \varrho_+^c, \varrho_+^d\right)}{\partial \varrho_+^c} = \beta\left[1 + \left(1-\delta^c\right)\mu_+^c\right] \tag{E.23}$$

$$\varrho_+^d : \mu^d = \beta\frac{\partial P\left(n_+^c, n_+^d, \varrho_+^c, \varrho_+^d\right)}{\partial \varrho_+^d} = \beta\left[-1 + \left(1-\delta^d\right)\mu_+^d\right] \tag{E.24}$$

$$v^x : -\kappa^x + \lambda^x q\left(\alpha^x\left(\rho_+^x\right)\right) + \mu^x q\left(\alpha^x\left(\rho_+^x\right)\right)\rho_+^x = 0,\ x = \{c, d\} \tag{E.25}$$

$$\rho_+^x : \lambda^x \frac{\partial q\left(\alpha^x\left(\rho_+^x\right)\right)}{\partial \rho_+^x} v^x + \mu^x \left[\frac{\partial q\left(\alpha^x\left(\rho_+^x\right)\right)}{\partial \rho_+^x} \rho_+^x + q\left(\alpha^x\left(\rho_+^x\right)\right)\right] v^x = 0,\ x = \{c, d\} \quad \text{(E.26)}$$

Lagrangian multiplier values $\{\mu^x\}_{x=c,d}$ are deduced from (E.23) and (E.24) as

$$\mu^c = \frac{1}{r + \delta^c}, \text{ and } \mu^d = -\frac{1}{r + \delta^d} \quad \text{(E.27)}$$

Lagrangian multiplier values $\{\lambda^x\}_{x=c,d}$ are deduced from (E.25) and (E.27)

$$\lambda^c = \frac{\kappa^c}{q\left(\alpha^c\left(\rho_+^c\right)\right)} - \frac{\rho_+^c}{r + \delta^c} \quad \text{(E.28)}$$

$$\lambda^d = \frac{\kappa^d}{q\left(\alpha^d\left(\rho_+^d\right)\right)} + \frac{\rho_+^d}{r + \delta^d} \quad \text{(E.29)}$$

Equation (E.21) and (E.22) give two expressions for λ^i

$$\lambda^i = \left(r + \delta^d\right)\lambda^d, \text{ and } \lambda^i = -\left(r + \delta^c\right)\lambda^c \quad \text{(E.30)}$$

The equality between these two expressions for λ^i given by (E.30) yields

$$\left(r + \delta^d\right)\lambda^d + \left(r + \delta^c\right)\lambda^c = 0 \quad \text{(E.31)}$$

and, for the values of λ^x given by (E.28) and (E.29), (E.31) becomes

$$\left(r + \delta^c\right)\frac{\kappa^c}{q\left(\alpha^c\left(\rho_+^c\right)\right)} + \left(r + \delta^d\right)\frac{\kappa^d}{q\left(\alpha^d\left(\rho_+^d\right)\right)} = \rho_+^c - \rho_+^d \quad \text{(E.32)}$$

The interpretation of the RHS term of (E.32) is similar to that of (7) for the social planner, except that the representative bank considers that average search costs on market x, namely $\kappa^x / q\left(\alpha^x\left(\rho^x\right)\right)$, because it takes as given the market tightness contrary to the social planner. The LHS of (E.32) is the private return for financial intermediation, also known as the net interest margin.

Posted interest rate strategies satisfy (39), or equivalently

$$\rho_+^x : \frac{\lambda^x}{\mu^x} + \rho_+^x = -\frac{q\left(\alpha^x\left(\rho_+^x\right)\right)}{\partial q\left(\alpha^x\left(\rho_+^x\right)\right) / \partial\left(\rho_+^x\right)},\ x = \{c, d\} \quad \text{(E.33)}$$

Using (E.25), (E.33) becomes

$$\rho_+^x : \frac{1}{\mu^x}\frac{\kappa^x}{q\left(\alpha^x\left(\rho_+^x\right)\right)} = -\frac{q\left(\alpha^x\left(\rho_+^x\right)\right)}{\partial q\left(\alpha^x\left(\rho_+^x\right)\right)/\partial\left(\rho_+^x\right)},\ x = \{c, d\} \tag{E.34}$$

and, with (E.27), (E.34) is finally

$$\left(r+\delta^c\right)\frac{\kappa^c}{q\left(\alpha^c\left(\rho_+^c\right)\right)} = -\frac{q\left(\alpha^c\left(\rho_+^c\right)\right)}{\partial q\left(\alpha^c\left(\rho_+^c\right)\right)/\partial\rho_+^c} \tag{E.35}$$

and

$$\left(r+\delta^d\right)\frac{\kappa^d}{q\left(\alpha^d\left(\rho_+^d\right)\right)} = \frac{q\left(\alpha^d\left(\rho_+^d\right)\right)}{\partial q\left(\alpha^d\left(\rho_+^d\right)\right)/\partial\rho_+^d} \tag{E.36}$$

Equations (E.20) and (E.35) give the equilibrium credit interest rate

$$\rho_p^c = z - \left(\frac{1-\varepsilon}{\varepsilon}\right)\left[\left(r+\delta^c\right)\frac{\kappa^c}{\overline{m}^c}\left(\alpha_p^c\right)^{1-\varepsilon} + \kappa^c\alpha_p^c\right] \tag{E.37}$$

Equations (E.12) and (E.36) give the equilibrium deposit interest rate

$$\rho_p^d = \rho^h + \left(\frac{1-\varepsilon}{\varepsilon}\right)\left(r+\delta^d\right)\frac{\kappa^d}{\overline{m}^d}\left(\alpha_p^d\right)^{1-\varepsilon} \tag{E.38}$$

E.4. Proof of Proposition 3

For the interest rate (E.38), the condition (E.4) determines the deposit market tightness

$$\alpha_p^d = \frac{\kappa^h}{\kappa^d}\left(\frac{\varepsilon}{1-\varepsilon}\right) \tag{E.39}$$

which is identical to the social planner solution, see (6). The net interest margin associated with (E.37) and (E.38) is

$$\begin{aligned}\rho_p^c - \rho_p^d &= z - \rho^h - \left(\frac{1-\varepsilon}{\varepsilon}\right)\left[\left(r+\delta^c\right)\frac{\kappa^c}{\overline{m}^c}\left(\alpha_p^c\right)^{1-\varepsilon} + \kappa^c\alpha_p^c\right] \\ &\quad - \left(\frac{1-\varepsilon}{\varepsilon}\right)\left(r+\delta^d\right)\frac{\kappa^d}{\overline{m}^d}\left(\alpha_p^d\right)^{1-\varepsilon}\end{aligned} \tag{E.40}$$

This expression is introduced into the condition (E.32) to get the credit market tightness

$$\begin{aligned}&\left(r+\delta^d\right)\frac{\kappa^d}{q\left(\alpha_p^d\right)} + \left(r+\delta^c\right)\frac{\kappa^c}{q\left(\alpha_p^c\right)} \\ &= z - \rho^h - \left(\frac{1-\varepsilon}{\varepsilon}\right)\left[\left(r+\delta^c\right)\frac{\kappa^c}{\overline{m}^c}\left(\alpha_p^c\right)^{1-\varepsilon} + \kappa^c\alpha_p^c\right] - \left(\frac{1-\varepsilon}{\varepsilon}\right)\left(r+\delta^d\right)\frac{\kappa^d}{\overline{m}^d}\left(\alpha_p^d\right)^{1-\varepsilon}\end{aligned} \tag{E.41}$$

or equivalently

$$\left(r+\delta^{c}\right)\frac{\kappa^{c}}{\overline{m}^{c}}\left(\alpha_{p}^{c}\right)^{1-\varepsilon}+\left(r+\delta^{d}\right)\frac{\kappa^{d}}{\overline{m}^{d}}\left(\alpha_{p}^{d}\right)^{1-\varepsilon}=\varepsilon\left(z-\rho^{h}\right)-\left(1-\varepsilon\right)\kappa^{c}\alpha_{p}^{c} \tag{E.42}$$

which is identical to the social planner solution, see (7).

F. Interbank Market Frictions

This Section solves the model with directed search by and interest rate posted by specialized banks.

F.1. Search and Interest Rate Posted by Specialized Banks

The FOCs associated to the value function (45) are

$$\varrho_{+}^{d}:\mu^{d}=\beta\sigma\frac{\partial P^{d}\left(\widetilde{n},\varrho_{+}^{d}\right)}{\partial\varrho_{+}^{d}}=\beta\sigma\left[-1+\left(1-\delta^{d}\right)\mu^{d}\right] \tag{F.1}$$

$$v^{d}:-\kappa^{d}+\lambda^{d}q\left(\alpha\left(\rho_{+}^{d}\right)\right)+\mu^{d}q\left(\alpha^{d}\left(\rho_{+}^{d}\right)\right)\rho_{+}^{d}=0 \tag{F.2}$$

$$\rho_{+}^{d}:\lambda^{d}\frac{\partial q\left(\alpha^{d}\left(\rho_{+}^{d}\right)\right)}{\partial\rho_{+}^{d}}v^{d}+\mu^{d}\left[\frac{\partial q\left(\alpha^{d}\left(\rho_{+}^{d}\right)\right)}{\partial\rho_{+}^{d}}\rho_{+}^{d}+q\left(\alpha^{d}\left(\rho_{+}^{d}\right)\right)\right]v^{d}=0 \tag{F.3}$$

Proceeding as in Section E, the deposit interest rate satisfies now

$$\left(r^{*}+\delta^{d}\right)\frac{\kappa^{d}}{q\left(\alpha^{d}\left(\rho_{+}^{d}\right)\right)}=\frac{q\left(\alpha^{d}\left(\rho_{+}^{d}\right)\right)}{\partial q\left(\alpha^{d}\left(\rho_{+}^{d}\right)\right)/\partial\left(\rho_{+}^{d}\right)} \tag{F.4}$$

The FOCs associated to the value function (46) are

$$\varrho_{+}^{c}:\mu^{c}=\beta\sigma\frac{\partial P^{c}\left(\widetilde{n},\varrho_{+}^{c}\right)}{\partial\varrho_{+}^{c}}=\beta\sigma\left[1+\left(1-\delta^{c}\right)\mu^{c}\right] \tag{F.5}$$

$$v^{c}:-\kappa^{c}+\lambda^{c}q\left(\alpha^{c}\left(\rho_{+}^{c}\right)\right)+\mu^{c}q\left(\alpha^{c}\left(\rho_{+}^{c}\right)\right)\rho_{+}^{c}=0 \tag{F.6}$$

$$\rho_{+}^{c}:\lambda^{c}\frac{\partial q\left(\alpha^{c}\left(\rho_{+}^{c}\right)\right)}{\partial\rho_{+}^{c}}v^{c}+\mu^{c}\left[\frac{\partial q\left(\alpha^{c}\left(\rho_{+}^{c}\right)\right)}{\partial\rho_{+}^{c}}\rho_{+}^{c}+q\left(\alpha^{c}\left(\rho_{+}^{c}\right)\right)\right]v^{c}=0 \tag{F.7}$$

Proceeding as in Section E, the deposit interest rate satisfies now

$$(r^* + \delta^c) \frac{\kappa^c}{q\left(\alpha^c\left(\rho^c_+\right)\right)} = -\frac{q\left(\alpha^c\left(\rho^c_+\right)\right)}{\partial q\left(\alpha^c\left(\rho^c_+\right)\right)/\partial\left(\rho^c_+\right)} \tag{F.8}$$

The value functions (45) and (46) are linear function of the variable $\widetilde{n}$, namely the mass of customers, which does not appear in the FOCs. Therefore, the search effort of banks differs according to their state. In old matches, the search effort is v^x to replace the $\delta^x \widetilde{n}$ lost customers

$$v^x = \frac{\delta^x \widetilde{n}}{q\left(\alpha^x\left(\rho^x_+\right)\right)} \tag{F.9}$$

given the matching probability $q\left(\alpha^x\left(\rho^x_+\right)\right)$, whereas in newly created matches, the search effort is v_0^x to find the $\widetilde{n}$ customers

$$v_0^x = \frac{\widetilde{n}}{q\left(\alpha^x\left(\rho^x_+\right)\right)} = v^x + \frac{\left(1-\delta^x\right)\widetilde{n}}{q\left(\alpha^x\left(\rho^x_+\right)\right)} \tag{F.10}$$

for $x = \{c, d\}$.

F.2. Financial Contract

The first order conditions of the Nash Solution defined by (44) are

$$\widetilde{\eta}\frac{1}{P^d(0,0)+\widetilde{\varrho}} + (1-\widetilde{\eta})\frac{-1}{P^c(0,0)-\widetilde{\varrho}} = 0 \tag{F.11}$$

with respect to $\widetilde{\varrho}$, and

$$\widetilde{\eta}\frac{\beta\sigma\frac{\partial P^d\left(\widetilde{n},\varrho^d_+\right)}{\partial\widetilde{n}} - \lambda^d}{P^d(0,0)+\widetilde{\varrho}} + (1-\widetilde{\eta})\frac{\beta\sigma\frac{\partial P^c\left(\widetilde{n},\varrho^c_+\right)}{\partial\widetilde{n}} - \lambda^c}{P^c(0,0)-\widetilde{\varrho}} = 0 \tag{F.12}$$

with respect to $\widetilde{n}$. Equation (F.11) can be rearranged as:

$$\widetilde{\varrho} = \widetilde{\eta}P^c(0,0) - (1-\widetilde{\eta})P^d(0,0) \tag{F.13}$$

Using (F.13), (F.12) becomes:

$$\beta\sigma\frac{\partial P^d\left(\widetilde{n},\varrho^d_+\right)}{\partial\widetilde{n}} + \beta\sigma\frac{\partial P^c\left(\widetilde{n},\varrho^c_+\right)}{\partial\widetilde{n}} = \lambda^d + \lambda^c \tag{F.14}$$

and, using (45) and (46), it becomes

$$\beta\sigma\left(1-\delta^{d}\right)\lambda^{d}+\beta\sigma\left(1-\delta^{c}\right)\lambda^{c}=\lambda^{d}+\lambda^{c} \tag{F.15}$$

and, using $r^{*}=(\beta\sigma)^{-1}-1$, it becomes

$$\left(r^{*}+\delta^{d}\right)\lambda^{d}+\left(r^{*}+\delta^{c}\right)\lambda^{c}=0 \tag{F.16}$$

For the values of the multipliers λ^{d} and λ^{d}, given by the FOCs (F.1)-(F.2) and (F.5)-(F.6), the optimal search efforts defined by (F.16) satisfy

$$\left(r^{*}+\delta^{d}\right)\frac{\kappa^{d}}{q\left(\alpha^{d}\left(\rho_{+}^{d}\right)\right)}+\left(r^{*}+\delta^{c}\right)\frac{\kappa^{c}}{q\left(\alpha^{d}\left(\rho_{+}^{c}\right)\right)}=\rho_{+}^{c}-\rho_{+}^{d} \tag{F.17}$$

To determine the equilibrium payment $\widetilde{\varrho}$, one need expressions for the value functions of specialized banks in a newly created match. The value functions of the deposit bank are

$$\begin{aligned} P^{d}(0,0) &= -\kappa^{d}v_{0}^{d}+\beta\left[\sigma P^{d}\left(\widetilde{n},\varrho_{+}^{d}\right)+(1-\sigma)P^{d}(0,0)\right] \\ &= -\frac{\kappa^{d}v_{0}^{d}}{1-\beta(1-\sigma)}+\frac{\beta\sigma}{1-\beta(1-\sigma)}P^{d}\left(\widetilde{n},\varrho_{+}^{d}\right) \end{aligned} \tag{F.18}$$

in a newly created match, and

$$\begin{aligned} P^{d}\left(\widetilde{n},\varrho^{d}\right) &= -\varrho^{d}-\kappa^{d}v^{d}+\beta\sigma P^{d}\left(\widetilde{n},\varrho_{+}^{d}\right)+(1-\sigma)\beta P^{d}(0,0) \\ &= -\frac{1}{1-\beta\sigma}\varrho^{d}-\frac{1}{1-\beta\sigma}\kappa^{d}v^{d}+\frac{(1-\sigma)\beta}{1-\beta\sigma}P^{d}(0,0) \end{aligned} \tag{F.19}$$

in an existing match. Introducing (F.19) into (F.18) gives

$$\left(\frac{1-\beta}{\beta\sigma}\right)P^{d}(0,0)=-\varrho^{d}-\frac{1-\beta\sigma}{\beta\sigma}\kappa^{d}v_{0}^{d}-\kappa^{d}v^{d} \tag{F.20}$$

The value functions of the credit bank are

$$\begin{aligned} P^{c}(0,0) &= -\kappa^{c}v_{0}^{c}+\beta\left[\sigma P^{c}\left(\widetilde{n},\varrho_{+}^{c}\right)+(1-\sigma)P^{c}(0,0)\right] \\ &= -\frac{\kappa^{c}v_{0}^{c}}{1-\beta(1-\sigma)}+\frac{\beta\sigma}{1-\beta(1-\sigma)}P^{c}\left(\widetilde{n},\varrho_{+}^{c}\right) \end{aligned} \tag{F.21}$$

in a newly created match, and

$$\begin{aligned} P^c(\widetilde{n}, \varrho^c) &= \varrho^c - \kappa^c v^c + \beta\left[\sigma P^c\left(\widetilde{n}, \varrho^c_+\right) + (1-\sigma) P^c(0,0)\right] \\ &= \frac{\varrho^c - \kappa^c v^c}{1-\beta\sigma} + \frac{\beta(1-\sigma)}{1-\beta\sigma} P^c(0,0) \end{aligned} \tag{F.22}$$

in an existing match. Introducing (F.22) into (F.21) gives

$$\left(\frac{1-\beta}{\beta\sigma}\right) P^c(0,0) = \varrho^c - \frac{1-\beta\sigma}{\beta\sigma}\kappa^c v^c_0 - \kappa^c v^c \tag{F.23}$$

Finally, (F.20) and (F.23) are introduced into (F.13) to get the expression $\widetilde{\varrho}$ given by (47).

F.3. Directed Search by Non-Financial Agents

The value functions are modified because of the probability $(1-\sigma)$ of separation on the interbank market. For households, the value functions are

$$D^m\left(\rho^d_+\right) = \rho^d_+ + \left(1-\delta^d\right)\left[\sigma\beta D^m\left(\rho^d_+\right) + (1-\sigma)\beta D^u\right] + \delta^d \beta D^u\left(\overline{\rho}^d_+\right) \tag{F.24}$$

and

$$D^u = \rho^h - \kappa^h + p\left(\alpha^d\left(\rho^d_+\right)\right)\left[\sigma\beta D^m\left(\rho^d_+\right) + (1-\sigma)\beta D^u\right] + \left[1 - p\left(\alpha^d\left(\rho^d_+\right)\right)\right]\beta D^u \tag{F.25}$$

For entrepreneurs, the value functions are

$$C^m\left(\rho^c_+\right) = z - \rho^c_+ + (1-\delta^c)\left[\sigma\beta C^m\left(\rho^c_+\right) + (1-\sigma)\beta C^u\right] + \delta^c\beta C^u \tag{F.26}$$

and

$$C^u = p\left(\alpha^c\left(\rho^c_+\right)\right)\left[\sigma\beta C^m\left(\rho^c_+\right) + (1-\sigma)\beta C^u\right] + \left[1 - p\left(\alpha^c\left(\rho^c_+\right)\right)\right]\beta C^u \tag{F.27}$$

The directed search behavior is now defined by

$$\frac{\partial p\left(\alpha^c\left(\rho^c_+\right)\right)}{\partial \rho^c_+}\frac{\rho^c_+}{p\left(\alpha^c\left(\rho^c_+\right)\right)} = -\frac{\rho^c_+}{z-\rho^c}\frac{1-\left[1-\delta^c - p\left(\alpha^c\left(\rho^c_+\right)\right)\right]\sigma\beta}{1-(1-\delta^c)\sigma\beta} \tag{F.28}$$

for entrepreneurs, and by

$$\frac{\partial q\left(\alpha^d\left(\rho^d_+\right)\right)}{\partial \rho^d_+}\frac{\rho^d_+}{q\left(\alpha^d\left(\rho^d_+\right)\right)} = \left(\frac{1-\varepsilon}{\varepsilon}\right)\left(\frac{\rho^d_+}{\rho^d_+ - \rho^h}\right) > 0 \tag{F.29}$$

for households. The new expression of the free entry condition for households is

$$\kappa^h = p\left(\alpha^d\left(\rho^d_+\right)\right)\frac{\rho^d_+ - \rho^h}{r^* + \delta^d} \tag{F.30}$$

F.4. Interest Rates and Tightness

Proceeding as in Section E, one gets the following expressions for the posted credit interest rate

$$\rho_+^c = z + \frac{q\left(\alpha^c\left(\rho_+^c\right)\right)}{\partial q\left(\alpha^c\left(\rho_+^c\right)\right)/\partial\rho_+^c}\left(\frac{1-\varepsilon}{\varepsilon}\right)\frac{1-\left[1-\delta^c-p\left(\alpha^c\left(\rho_+^c\right)\right)\right]\sigma\beta}{1-\left(1-\delta^c\right)\sigma\beta} \tag{F.31}$$

and for the posted deposit interest rate

$$\rho_+^d = \rho^h + \frac{q\left(\alpha^d\left(\rho_+^d\right)\right)}{\partial q\left(\alpha_+^d\left(\rho_+^d\right)\right)/\partial\rho_+^d}\left(\frac{1-\varepsilon}{\varepsilon}\right) \tag{F.32}$$

Equations (F.8) and (F.31) give the equilibrium credit interest rate

$$\rho_s^c = z - \left(\frac{1-\varepsilon}{\varepsilon}\right)\left[\left(r^*+\delta^c\right)\frac{\kappa^c}{\overline{m}^c}\left(\alpha_s^c\right)^{1-\varepsilon}+\kappa^c\alpha_s^c\right] \tag{F.33}$$

Equations (F.4) and (F.32) give the equilibrium deposit interest rate

$$\rho_s^d = \rho^h + \left(\frac{1-\varepsilon}{\varepsilon}\right)\left(r^*+\delta^d\right)\frac{\kappa^d}{\overline{m}^d}\left(\alpha_s^d\right)^{1-\varepsilon} \tag{F.34}$$

For the interest rate (F.34), the condition (F.30) determines the deposit market tightness

$$\alpha_s^d = \frac{\kappa^h}{\kappa^d}\left(\frac{\varepsilon}{1-\varepsilon}\right) \tag{F.35}$$

The net interest margin associated with (F.33) and (F.34) is

$$\rho_s^c - \rho_s^d = z - \rho^h - \left(\frac{1-\varepsilon}{\varepsilon}\right)\left[\left(r^*+\delta^c\right)\frac{\kappa^c}{\overline{m}^c}\left(\alpha_s^c\right)^{1-\varepsilon}+\kappa^c\alpha_s^c\right] - \left(\frac{1-\varepsilon}{\varepsilon}\right)\left(r^*+\delta^d\right)\frac{\kappa^d}{\overline{m}^d}\left(\alpha_s^d\right)^{1-\varepsilon} \tag{F.36}$$

This expression is introduced into the condition (F.17) to get the credit market tightness

$$\begin{aligned}&\left(r^*+\delta^d\right)\frac{\kappa^d}{q\left(\alpha^d\left(\rho_+^d\right)\right)}+\left(r^*+\delta^c\right)\frac{\kappa^c}{q\left(\alpha^d\left(\rho_+^c\right)\right)} \\ =\;& z-\rho^h-\left(\frac{1-\varepsilon}{\varepsilon}\right)\left[\left(r^*+\delta^c\right)\frac{\kappa^c}{\overline{m}^c}\left(\alpha_s^c\right)^{1-\varepsilon}+\kappa^c\alpha_s^c\right]-\left(\frac{1-\varepsilon}{\varepsilon}\right)\left(r^*+\delta^d\right)\frac{\kappa^d}{\overline{m}^d}\left(\alpha_s^d\right)^{1-\varepsilon}\end{aligned} \tag{F.37}$$

or equivalently

$$\left(r^*+\delta^c\right)\frac{\kappa^c}{\overline{m}^c}\left(\alpha_s^c\right)^{1-\varepsilon}+\left(r^*+\delta^d\right)\frac{\kappa^d}{\overline{m}^d}\left(\alpha_s^d\right)^{1-\varepsilon}=\varepsilon\left(z-\rho^h\right)-\left(1-\varepsilon\right)\kappa^c\alpha_s^c \tag{F.38}$$

G. Notations (Not intended for publication)

- General notations

y^x is the parameter or variable y on the x market where $x = \{c, d\}$ denotes the credit market (c) and the deposit market (d)

y_+ is the tomorrow value of the state variable y

- Population

$\overline{n}^d > 0$: total population of households

$u^d > 0$: households that search for a bank on the deposit market (unmatched)

$n^d > 0$: households matched with a bank

$o^d > 0$: unmatched households that do not search (no-participating)

$\overline{n}^c > 0$: total population of entrepreneurs

$u^c > 0$: entrepreneurs that search for a bank on the credit market

$n^c > 0$: entrepreneurs matched with a bank

$[0, 1]$: size of the continuum of banks

- Costs

$\kappa^x > 0$: bank's search cost on the market $x = \{c, d\}$

$\kappa^h > 0$: household's search cost on the deposit market

- Production technologies

$\rho^h \geq 0$: domestic production of final good by one household

$z > 0$: production of final good by one entrepreneur

- Interest rates

$\beta = 1/(1+r) \in]0,1[$: the discount factor (identical for households, entrepreneurs, and banks)

$r = (1/\beta - 1) > 0$: interest rate to discount for time-preference

$r^* = 1/(\sigma\beta) - 1 > 0$: interest rate to discount for time-preference and exogenous separation rate on the interbank market

$\rho^x > 0$: interest rates on market $x = \{c, d\}$

$\rho^x_+ > 0$: posted interest rates on market $x = \{c, d\}$ for future periods

$\varrho^x > 0$: predetermined interests on market $x = \{c, d\}$ when interest rates are posted

$\widetilde{\varrho} > 0$: interbank market payment from the credit bank to the deposit bank

- Additional parameters and variables

$\delta^x \in]0,1[$: exogenous bank exit rates of the non-financial agents on market $x = \{c, d\}$

$\eta^x \in]0,1[$: Nash bargaining power of non-financial agents on market $x = \{c, d\}$

$\eta^b \in]0,1[$: Nash bargaining power of banks when bargaining is multilateral

$\widetilde{\eta} \in]0,1[$: Nash bargaining power of specialized banks in deposit activity

λ^x : Lagrangian multipliers associated with the dynamic constraint on n^x for $x = \{c, d\}$

μ^x : Lagrangian multipliers associated with the specific dynamic constraint on ϱ^x for $x = \{c, d\}$

λ^i : Lagrangian multiplier associated with the bank's balance sheet constraint $n^c \leq n^d$

$(1-\sigma) \in]0,1[$: exogenous separation rate of specialized banks

- Value functions

D^s : value function for households associated with the $s = \{h, u, m\}$ states, where h is for outside the banking sector, u for unmatched and searching for a bank, and m for matched with a bank

C^s : value function for entrepreneurs associated with the $s = \{u, m\}$ states, where u is for unmatched and searching for a bank, and m for matched with a bank

O : value function for the social planner

P : value function for the bank when interest rates are posted

P^x : value function for the bank specialized in market $x = \{c, d\}$ when interest rates are posted

B : value function for the bank when interest rates are bargained

ΔB^x : bank's surplus in the Nash bargaining program on market $x = \{c, d\}$

- The matching process

$\overline{m}^x > 0$: scale parameter of the matching technology on market $x = \{c, d\}$

$\varepsilon \in]0, 1[$: elasticity parameter of the matching functions

$\alpha^x = v^x / u^x$: market tightness of the x market for $x = \{c, d\}$

$p(\alpha^x)$: matching probability of non-financial agents on the x market

$q(\alpha^x)$: matching probability of search effort for banks on the x market