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Banking Leverage Procyclicality: a Theoretical Model Introducing Currency Diversification

Justine Pedrono

Highlights

- This paper offers the first theoretical model investigating the procyclicality of banking leverage relative to the currency diversification of banks' balance sheet.
- Using a contracting model between a representative bank and its creditor, we show that currency diversification still allows banks to follow a Value-at-Risk rule.
- Currency diversification changes banks' leverage adjustment in response to economic fluctuation. The impact on leverage procyclicality then depends on the type of shock.
- Introducing a floating exchange rate regime increases the fund-raising capacity of banks when shocks are not symmetric.

Abstract

The brutal adjustments to global banks' balance sheets in the wake of the recent economic crisis have rekindled interest in the procyclicality of banking leverage. This paper extends Adrian and Shin (2014) by allowing banks to hold assets and liabilities denominated in foreign currency. It investigates the procyclicality of banking leverage relative to the currency diversification of banks' balance sheet. Therefore, it provides a complete theoretical framework that explains heterogeneity in financial cycles when focusing on currency exposures. Our results show that the Value-at-Risk rule followed by banks is still validated when currency diversification is introduced. However, currency diversification changes leverage procyclicality where the decrease or increase in leverage procyclicality relative to the home economy will depend on the type of shock. To the extent that changes in state of nature are asymmetric, currency diversification of assets associated with floating exchange rate regime then increases the risk-taking capacity of banks.

Keywords

Financial Intermediary, Leverage, Procyclicality, Currency, Diversification, Value-at-Risk, Exchange Rate.

JEL

F36, G15, G21, G32.

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RESEARCH AND EXPERTISE ON THE WORLD ECONOMY

Banking leverage procyclicality: a theoretical model introducing currency diversification[1]

Justine Pedrono*

1. Introduction

The development experienced by banks pre-crisis, followed by the crisis downturn, have recently drawn attention to bank leverage adjustments. According to Adrian and Shin [2014], the leverage of banks, defined as the ratio of total assets to equity, is procyclical: it rises in good times and falls in downturns. This procyclicality has two sources. First, banks' balance sheets are marked-to-market. Thus, an improvement in economic activity increases their net worth. Second, banks are active in the management of their balance sheets: their equity remaining constant, they reallocate the increase in their net worth to additional borrowing and investment. The leverage therefore increases. Figure 1 illustrates balance sheet adjustment following an improvement in economic activity.

As banks are active in the management of their balance sheet, their behavior is compatible with a Value at Risk (VaR) rule. Adrian and Shin [2014] build a micro-founded model which links leverage to the VaR rule. Banks adjust their balance sheets to maintain a given probability of failure.

Empirically, Adrian and Shin [2008], Kalemli-Ozcan et al. [2012], Baglioni et al. [2013] find that bank's leverage is procyclical especially for investment banks. When measuring leverage procyclicality with the correlation between the growth rate of asset and the growth rate of leverage, figure 2 supports this conclusion at the aggregate level for the Euro area and for

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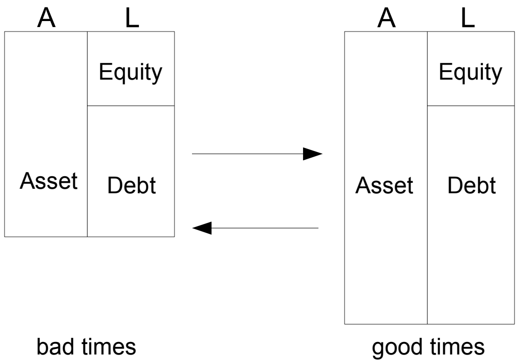
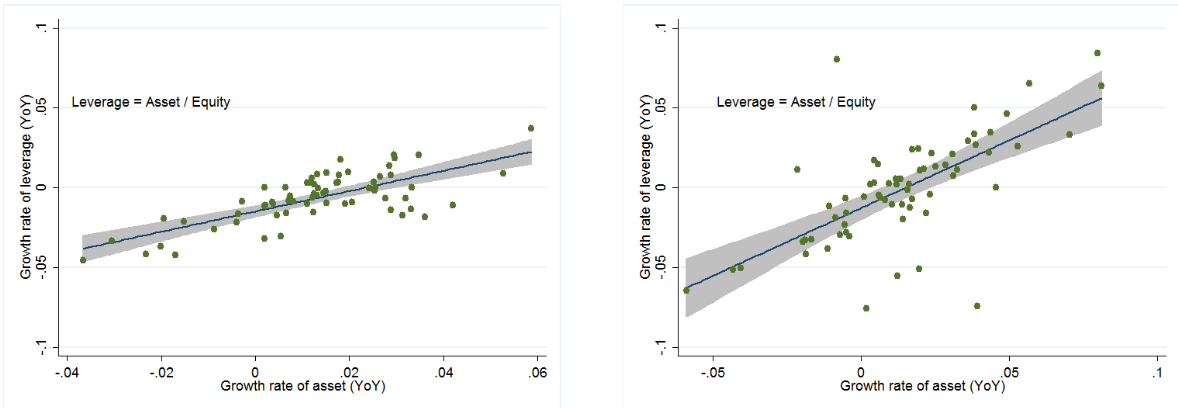


Figure 1 – Procyclical leverage - Adrian and Shin (2014)



(a) Banks located in the euro area

(b) Banks located in France

Figure 2 – **Procyclical leverage of banks:** Data are aggregated and cover the period Q1.1997-Q1.2014 (quarterly data). Each dot represents a quarter.
Sources: ECB, Banque de France, own calculation.

France. Although banks located in France show more procyclical leverage than banks located in the euro area, the procyclicality of banks located in France is also more dispersed than the procyclicality of banks located in euro area, indicating some heterogeneity in leverage procyclicality.

According to Kalemli-Ozcan et al. [2012], European investment banks show less procyclical leverage than US banks. This heterogeneity may reflect differences in the composition of banks' balance sheets. As banks use their collateral to raise funds, the composition of their collateral impacts leverage procyclicality. One major issue in this respect may be the currency denomination of the assets, which has not been incorporated in theoretical and empirical analyses on leverage procyclicality.

Adrian and Shin [2014] use a contracting model between a representative bank and its creditor, where the latter faces a default risk resulting from the risky investment made by the bank. The leverage then depends on the domestic state of nature. Their model micro-founds the VaR Rule but excludes any currency diversification. More recently, Bruno and Shin [2015] introduce a cross-border network with an exogenous exchange rate, a global bank, a regional bank and a local firms. Both the global and the regional bank carry out their financial operations in foreign currency, therefore there is no currency diversification in their balance sheets. In contrast, the local firm invests in local currency and raises debt from the regional bank in foreign currency. Thus, currency risk is only borne by the local firm. Such setting addresses part of the currency mismatch issue. However, as evidenced by Baba et al. [2009], Borio and Disyatat [2011], Shin [2012], McGuire and Von Peter [2012], global banks in advanced economies have developed large international strategies in their funding and financial portfolios, especially for European banks before the financial crisis.[2] Such international strategies induce some degree of currency diversification of both assets and liabilities. Figure 3 gives a breakdown by currency of external banking positions based on BIS Data. Focusing on G7 countries, the US dollar and the euro are the two major currencies used by reporting banks for both sides of the balance sheet.[3] Thus, banks' balance sheets are diversified in terms of currencies. Because they affect the value of banks' collateral, exchange rate variations should then be directly incorporated in the analysis of leverage dynamics through banks' balance sheet.

A flexible exchange rate interacts with other channels of leverage adjustment since it is correlated with asset returns. For instance, an appreciation of the US dollar is associated to an unexpected rise in US returns.[4] When the US economy is booming, foreign global banks see their assets denominated in US dollar revalued both due to higher returns and to the appreciation of the dollar. Therefore, they adjust their leverage upward. This channel is not taken into account by Bruno and Shin [2015]. Introducing currency diversification and endogenous exchange rate provides a complete theoretical framework that might explain

[2]As highlighted by Borio and Disyatat [2011], Baba et al. [2009], McGuire and Von Peter [2012], European banks were largely involved in US money markets before 2008.

[3]See also Pedrono [2015] for detailed stylized facts on currency diversification and banks located in France.

[4]Using intraday data, Kearns and Manners [2006] show that a positive interest rate shock in advanced economies leads to an appreciation of their currency. Alternatively, Ehrmann et al. [2011] use a structural VAR with daily data from 1988 to 2004. They show that a euro appreciation of 10 % is estimated to induce an increase in euro area financial markets of 5.7%. They also document an inverse causality where an increase of 100bp in euro area short rates leads to a 5.69% euro appreciation.

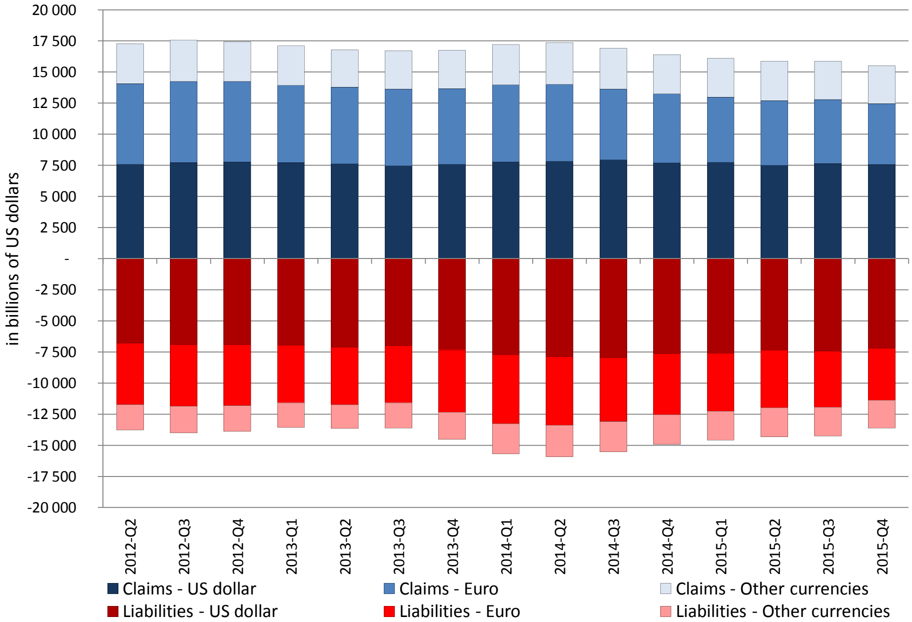


Figure 3 – Currency breakdown of cross-border claims and liabilities: reporting banks from G7 countries (Canada, France, Germany, Italy, Japan, the United Kingdom and the United States). Source: BIS, locational banking statistics, own calculation.

Kalemli-Ozcan et al. [2012] empirical results of less leverage procyclicality in Europe than in the United States, since European banks react both to the domestic and to the foreign asset cycle.

We extend the Adrian and Shin [2014] model by introducing a second currency of denomination on both sides of the balance sheet. The bank can borrow and invest in two different currencies: a domestic currency which is the currency of the bank's equity, and a foreign currency. The bank's balance sheet is expressed in domestic currency, which implies a conversion of foreign assets and liabilities. Two exchange rate regimes are successively studied: a fixed regime and a floating regime, where the exchange rate depends on the relative state of nature in the two issuing countries.[5]

One important result from Adrian and Shin [2014] concerns the VaR rule. As banks follow a VaR rule, they adjust their balance sheet in order to maintain a constant probability of default. Introducing currency diversification does not affect the VaR rule or the mechanism behind the VaR rule. However, depending on the type of shocks and on the exchange rate regime, balance sheet adjustment will be affected by the degree of currency diversification. Three

[5]In contrast to Bruno and Shin [2015], the exchange rate here is directly linked to the relative state of nature.

specific types of shocks are studied: (i) a symmetric change in states of nature (domestic and foreign) that does not affect the exchange rate; (ii) an anti-symmetric change in states of nature - or a change affecting the states of nature in an opposite manner - where the domestic economy is positively impacted while the foreign economy is negatively impacted; (iii) and an asymmetric change in states of nature whereby both economies are affected in the same direction, but one more strongly than the other.

A positive change in the home country state of nature induces a reaction of increased leverage of the home bank. If the change is symmetric across countries, currency diversification of the balance sheet does not modify the extent of the leverage procyclical reaction, whatever the exchange rate regime. If the change in states of nature is anti-symmetric, the expected return on foreign assets decreases while the expected return on domestic assets increases. Therefore, the total risk in bank's portfolio, covering both domestic and foreign assets, increases compared to a portfolio composed of domestic assets only. Assuming that exchange rate is fixed, we get a decrease in leverage. With a floating exchange rate, the home currency appreciates, so the weight of the domestic assets in the portfolio increases: the decrease in leverage is less important. Finally, if the change in states of nature is asymmetric with better conditions domestically, procyclicality diminishes with currency diversification.

The rest of the paper is organized as follows. Section 2 introduces the currency diversification in the Adrian and Shin [2014] framework. Section 3 develops the utility functions of agents. Two main constraints are derived from utility maximization. Section 4 defines the VaR rule and the reaction in terms of leverage to three changes in states of nature. Section 5 concludes and discusses some policy implications.

2. Currency diversification

Like in Adrian and Shin [2014], the model is based on a representative bank's balance sheet. The bank invests in assets and raises funds from its creditor. Here, though, there are two currency denominations for assets and debts, corresponding to two different countries (domestic and foreign). The economic states of nature corresponding to each economy are known publicly and determine the distribution of asset returns.

There are two periods T=0,1. Knowing the state of nature and the distribution of returns, the bank and the creditor agree on the amount to be reimbursed at T=1 in order to satisfy the VaR rule. This amount therefore defines the level of debt the bank is able to raise at T=0. The creditor incurs a risk of default resulting from the risk involved on the asset side of the bank's balance sheet.

2.1. The accounting framework

The representative bank is domestic in the sense that its equity and its balance sheet are in domestic currency. The bank is risk neutral and equity E is exogenous.[6] The second agent is the creditor of the bank, generally a Money Market Fund or another investment bank. The creditor lends money to the bank in both currencies (domestic and foreign). The creditor is also risk neutral. The exchange rate S is defined as the number of domestic units per unit of foreign currency.

At T=0, the bank raises funds backed by collateral in domestic and foreign currency (A and $A^\star$, respectively). Total assets expressed in domestic currency are equal to $A + SA^\star$. We denote by a the share of assets in domestic currency and $(1-a)$ the share of assets in foreign currency. a will vary depending on S. In this section, we consider S as fixed. Section 4 covers the case of a flexible exchange rate regime. Funds are in domestic and in foreign currency (D and $D^\star$, respectively). Thus, total funding from the creditor expressed in domestic currency is equal to $D+SD^\star$. This debt is defaultable, implying that the creditor receives a defaultable debt claim at T=0.

At T=1, the bank receives a total expected return from its investments $a(1+\bar{r}) + (1-a)(1+\bar{r}^\star)$, where $\bar{r}$ and $\bar{r}^\star$ are the expected returns from the domestic and the foreign asset, respectively. Returns depend on the state of nature specific to each currency area, θ and $\theta^\star$, respectively. θ and $\theta^\star$ are known publicly from T=0 and they do not change between the two periods. At T=1, the bank also reimburses its domestic and foreign debts, $\bar{D}$ and $S\bar{D}^\star$ respectively. As θ and $\theta^\star$ are known for the two periods, there is no macroeconomic risk. It is assumed that $\bar{D} > D$ and $S\bar{D}^\star > SD^\star$ to remunerate the creditor for the default risk.

The bank's balance sheet at each period are given in table 1 where $\bar{E}$ is the equity at notional value.

[6] An exogenous equity is in line with the theory of procyclical leverage put forward by Shin.

T=0, at market value: Assets	Liabilities
A	E
$SA^\star$	D
	$SD^\star$

T=1, at notional value: Assets	Liabilities
$(1+\bar{r})A$	$\bar{E}$
$(1+\bar{r}^\star)SA^\star$	$\bar{D}$
	$S\bar{D}^\star$

Table 1 – Bank's balance sheet at T=0 and T=1

2.2. Leverage

Four debt ratios are defined relative to each funding currency and value. The ratios of the market value of debt to the market value of assets are:

$$d = \frac{D}{A+SA^\star} \quad \text{and} \quad d^\star = \frac{SD^\star}{A+SA^\star} \tag{1}$$

Alternatively, the corresponding ratios of notional values of debt to total assets at the market value are:

$$\bar{d} = \frac{\bar{D}}{A+SA^\star} \quad \text{and} \quad \bar{d}^\star = \frac{S\bar{D}^\star}{A+SA^\star} \tag{2}$$

$\bar{E}$ is the equity at the notional value that sets the two sides of the balance sheet equal. The bank is expected to make profits such that $E < \bar{E}$ and $a(1+\bar{r})+(1-a)(1+\bar{r}^\star) > (\bar{d}+\bar{d}^\star)$.

The leverage λ is defined as the ratio of total assets to equity, at market value:

$$\lambda = \frac{A+SA^\star}{E} = \frac{A+SA^\star}{(A+SA^\star)-(D+SD^\star)} = \frac{1}{1-(d+d^\star)} \tag{3}$$

2.3. Investment strategy

The bank makes an indivisible choice between two types of portfolio. Each portfolio is composed of an asset in domestic currency and an asset in foreign currency. The weight of each type of asset is given by a and $(1-a)$. The portfolio's distribution comes from a mixture distribution of the two asset return distributions. As each asset return follows a General Extreme Value (GEV) distribution, the portfolio's return is also defined by a GEV distribution. The first portfolio is a "good" portfolio with a total expected return of $[ar_H + (1-a)r_{H^\star}]$, where r_H denotes the expected return from the good domestic asset and $r_{H^\star}$ the expected return from the good foreign asset. The second portfolio is not as good. Its total expected return $[ar_L + (1-a)r_{L^\star}]$ is reduced through a parameter k ($k > 0$) and its volatility is increased by a parameter m ($m > 1$) compared to the good portfolio.[7] The cumulative distribution of the good asset return in domestic currency, the cumulative distribution of the good asset return in foreign currency, the cumulative distribution of the bad asset return in domestic currency, and the cumulative distribution of the bad asset return in foreign currency are the following, where θ, σ and ξ are respectively the location parameter, the scale parameter and the shape parameter:[8]

$$
\begin{aligned}
F_H(z) &= exp\left\{-\left(1+\xi\left(\frac{z-\theta}{\sigma}\right)\right)^{-\frac{1}{\xi}}\right\} \\
F_{H^\star}(z) &= exp\left\{-\left(1+\xi\left(\frac{z-\theta^\star}{\sigma}\right)\right)^{-\frac{1}{\xi}}\right\} \\
F_L(z) &= exp\left\{-\left(1+\xi\left(\frac{z-(\theta-k)}{\sigma m}\right)\right)^{-\frac{1}{\xi}}\right\} \\
F_{L^\star}(z) &= exp\left\{-\left(1+\xi\left(\frac{z-(\theta^\star-k)}{\sigma m}\right)\right)^{-\frac{1}{\xi}}\right\}
\end{aligned} \tag{4}
$$

Domestic and foreign assets within each category then differ only in their location parameters - θ and $\theta^\star$ respectively - implying similar economies.

Using a mixture distribution, the Cumulative Distribution Function (CDF) of total return

[7]Introducing an investment choice enables a contract between the creditor and the bank to be modeled, as in Holmström and Tirole [1997].

[8]The distribution function of a real-valued random variable Z is given by $F(z) = P(Z \leq z)$. By adding location and scale parameters θ and σ like in GEV models, then the distribution function of the real-valued random variable Z is given by:

$$P(Z \leq (z-\theta)/\sigma) = P(Z\sigma + \theta \leq z) = F_{\theta,\sigma}(z)$$

Because all distribution functions include location and scale parameters specific to each asset, we simplify the notation of the full statistical model with $F(z)$ instead of $F_{\theta,\sigma}(z)$. See Reiss and Thomas [2007] for more details on GEV distributions.

when the bank invests in the good portfolio is : [9]

$$F_{H,H^\star}(z) = a\ exp\left\{-\left(1+\xi\left(\frac{z-\theta}{\sigma}\right)\right)^{-\frac{1}{\xi}}\right\} + (1-a)\ exp\left\{-\left(1+\xi\left(\frac{z-\theta^\star}{\sigma}\right)\right)^{-\frac{1}{\xi}}\right\} \tag{5}$$

If the bank invests in the bad portfolio, the CDF is defined by:

$$F_{L,L^\star}(z) = a\ exp\left\{-\left(1+\xi\left(\frac{z-(\theta-k)}{\sigma m}\right)\right)^{-\frac{1}{\xi}}\right\} + (1-a)\ exp\left\{-\left(1+\xi\left(\frac{z-(\theta^\star-k)}{\sigma m}\right)\right)^{-\frac{1}{\xi}}\right\}$$

Thus, the total expected return of the portfolio depends on the state of nature in the domestic country (θ) and in the foreign one ($\theta^\star$).

The CDF allows us to define the probability of default α when the bank invests in the good portfolio. Default appears if the realized total return falls below the total debt ratio at the notional value ($(\bar{d}+\bar{d}^\star) \geq z$). Thus, the probability of default α is defined by the cumulative distribution function such that:[10]

$$\begin{aligned}\alpha(\bar{d}+\bar{d}^\star) &= F_{H,H^\star}(\bar{d}+\bar{d}^\star)\\ &= a\ exp\left\{-\left(1+\xi\left(\frac{(\bar{d}+\bar{d}^\star)-\theta}{\sigma}\right)\right)^{-\frac{1}{\xi}}\right\}\\ &\quad + (1-a)\ exp\left\{-\left(1+\xi\left(\frac{(\bar{d}+\bar{d}^\star)-\theta^\star}{\sigma}\right)\right)^{-\frac{1}{\xi}}\right\}\end{aligned} \tag{6}$$

Since the creditor is uninsured, he/she holds a defaultable debt claim with respect to the funds lent to the bank at T=0. According to Merton [1974], the value of this defaultable debt claim with strike price ($\bar{D}+S\bar{D}^\star$) can be divided into two components: cash ($\bar{D}+S\bar{D}^\star$) and a short position on a put option π. Thereby, the value of a defaultable debt claim is lower than its expected payoff ($\bar{D}+S\bar{D}^\star$) because of its induced risk. Since the risk differs between the two types of portfolio, the put option is specific to each investment choice.

[9] This new framework using a mixture distribution is still compatible with a Second Order Stochastic Dominance, as in the reference model.

[10] Alternatively, the probability of default when the bank invests in the "less good" portfolio can be defined through $F_{L,L^\star}(\bar{d}+\bar{d}^\star)$. However, we focus on the good portfolio since the contract between the bank and its creditor leads to this portfolio in section 3.

If the bank invests in the good portfolio, we obtain the following put option price:[11]

$$\pi_{H,H^\star}(\bar{D}+S.\bar{D}^\star, A+SA^\star) = (A+SA^\star).\pi_{H,H^\star}(\bar{d}+\bar{d}^\star, 1) \equiv (A+SA^\star).\pi_{H,H^\star}(\bar{d}+\bar{d}^\star)$$

If the bank invests in the bad portfolio, the price of the put option is:

$$\pi_{L,L^\star}(\bar{D}+S.\bar{D}^\star, A+SA^\star) = (A+SA^\star).\pi_{L,L^\star}(\bar{d}+\bar{d}^\star, 1) \equiv (A+SA^\star).\pi_{L,L^\star}(\bar{d}+\bar{d}^\star)$$

3. Agents' participation constraints

3.1. Creditor's incentive constraint

The creditor of the bank is risk neutral. He maximizes his utility U^C defined as his total net expected payoff. His net expected payoff is the difference between the value of his defaultable debt claim and the total funds provided to the bank where:

$$\text{Defaultable debt claim} = (\bar{D}+S\bar{D}^\star) - (A+SA^\star)\pi(\bar{d}+\bar{d}^\star)$$

If the bank invests in the good portfolio, the net expected payoff is the following:

$$\begin{aligned} U^C_{H,H^\star}(A+SA^\star) &= (\bar{D}+S\bar{D}^\star) - (A+SA^\star)\pi_{H,H^\star}(\bar{d}+\bar{d}^\star) - (D+SD^\star) \\ &= (A+SA^\star)\left[(\bar{d}+\bar{d}^\star) - \pi_{H,H^\star}(\bar{d}+\bar{d}^\star) - (d+d^\star)\right] \end{aligned} \quad (7)$$

The requirement that utility is equal to or higher than 0 provides the first Participation Compatibility (PC) constraint of the creditor. This constraint binds in the optimal contract:

$$0 \leq (\bar{d}+\bar{d}^\star) - \pi_{H,H^\star}(\bar{d}+\bar{d}^\star) - (d+d^\star) \quad (8)$$

$$(d+d^\star) = (\bar{d}+\bar{d}^\star) - \pi_{H,H^\star}(\bar{d}+\bar{d}^\star) \quad \text{(PC)}$$

[11] The price of the put option depends on the total amount reimbursed at the end of the period - $\bar{D}+S.\bar{D}^\star$ - and on the total value of assets $A+SA^\star$. Assuming that there is a constant returns to scale of option price because of competitive markets, we can recover the value of the option on total portfolio $A+SA^\star$ with strike price$\bar{D}+S\bar{D}^\star$ by bundling together $A+SA^\star$ options on one dollar's worth of portfolio with strike price $(\bar{D}+S\bar{D}^\star)/(A+SA^\star)$.

Similarly for an investment in the bad portfolio:

$$\begin{aligned} U^C_{L,L^\star}(A+SA^\star) &= (A+SA^\star)\left[(\bar{d}+\bar{d}^\star)-\pi_{L,L^\star}(\bar{d}+\bar{d}^\star)-(d+d^\star)\right] \\ (d+d^\star) &= (\bar{d}+\bar{d}^\star)-\pi_{L,L^\star}(\bar{d}+\bar{d}^\star) \end{aligned} \quad (9)$$

The PC constraints define the total debt ratio at market value relative to the total debt ratio at notional value. The latter should be large enough to form an incentive for the creditor to participate. The higher the reimbursement offered by the bank, the more the creditor is tempted to lend money at T=0. In this form, the incentive does not depend directly on the portfolio return specifications.

3.2. Bank's incentive constraint

As the bank is risk neutral, it also maximizes its expected utility U^B defined as its total net expected payoff. The introduction of a second investment currency changes the composition of the bank's net expected payoff $U^B_{H,H^\star}$. In this framework, returns come from assets both in domestic and in foreign currency. Thus the net expected payoff when the bank invests in the good portfolio is equal to:

$$\begin{aligned} U^B_{H,H^\star} &= A.r_H + SA^\star r_{H^\star} + (D+S.D^\star) - (\bar{D}+S.\bar{D}^\star) + (A+SA^\star).\pi_{H,H^\star}(\bar{d}+\bar{d}^\star) \\ &= (A+SA^\star)\left[a.r_H + (1-a)r_{H^\star} + (d+d^\star) - (\bar{d}+\bar{d}^\star) + \pi_{H,H^\star}(\bar{d}+\bar{d}^\star)\right] \end{aligned} \quad (10)$$

When the bank invests in the bad portfolio the net expected payoff is equal to:

$$\begin{aligned} U^B_{L,L^\star} &= A.r_L + SA^\star r_{L^\star} + (D+S.D^\star) - (\bar{D}+S.\bar{D}^\star) + (A+SA^\star).\pi_{L,L^\star}(\bar{d}+\bar{d}^\star) \\ &= (A+SA^\star)\left[a.r_L + (1-a)r_{L^\star} + (d+d^\star) - (\bar{d}+\bar{d}^\star) + \pi_{L,L^\star}(\bar{d}+\bar{d}^\star)\right] \end{aligned} \quad (11)$$

Assuming that $U^B_{H,H^\star} \geq U^B_{L,L^\star}$ we get the Incentive Compatibility (IC) constraint:

$$a(r_H - r_L) + (1-a)(r_{H^\star} - r_{L^\star}) \geq \pi_{L,L^\star}(\bar{d}+\bar{d}^\star) - \pi_{H,H^\star}(\bar{d}+\bar{d}^\star) \quad (12)$$

Referring to equation (4), the expected return differentials $(r_H - r_L)$ and $(r_{H^\star} - r_{L^\star})$ are equal and independent on economic conditions.[12] Thus, the left hand side (lhs) of the IC

[12] See the appendix.

constraint defined in equation (12) can be simplified, as if the bank only held assets in the domestic currency.

$$r_H - r_L \geq \Delta\pi(\bar{d} + \bar{d}^\star) \tag{13}$$
$$\text{Where}: \ \Delta\pi(\bar{d} + \bar{d}^\star) = \pi_{L,L^\star}(\bar{d} + \bar{d}^\star) - \pi_{H,H^\star}(\bar{d} + \bar{d}^\star)$$
$$r_H - r_L = r_{H^\star} - r_{L^\star}$$

The IC constraint simplified in equation (13) stipulates that there is a solution $(\bar{d} + \bar{d}^\star)$ that satisfies this inequality. The unique solution illustrated in figure 4 comes from the Second Order Stochastic Dominance (SOSD) between the two mixture distributions and the differential in volatility. The surface area $\Delta\pi(z)$ increases until $F_{H,H^\star}(z) = F_{L,L^\star}(z)$ and decreases after the junction. As shareholders receive returns, $(\bar{d} + \bar{d}^\star) < a(1 + \bar{r}) + (1 - a)(1 + \bar{r}^\star)$, there is a unique solution $\bar{z} = (\bar{d} + \bar{d}^\star)$ which satisfies the IC constraint.

$$r_H - r_L = \Delta\pi(\bar{d} + \bar{d}^\star) \tag{IC}$$

As in Adrian and Shin [2014], the IC constraint also represents the moral hazard trade-off from Holmström and Tirole [1997]. The lhs of IC represents the bank's private benefit from investing in the good portfolio while the right hand side (rhs) is equal to the private benefit from investing in the bad portfolio (e.g. low effort in the moral hazard model of Holmström and Tirole [1997]). With the added PC constraint from the creditor, the bank necessarily invests in the good portfolio where the put option induces lower prices. However, additional assumptions are needed to obtain a closed form solution for $(\bar{d} + \bar{d}^\star)$.

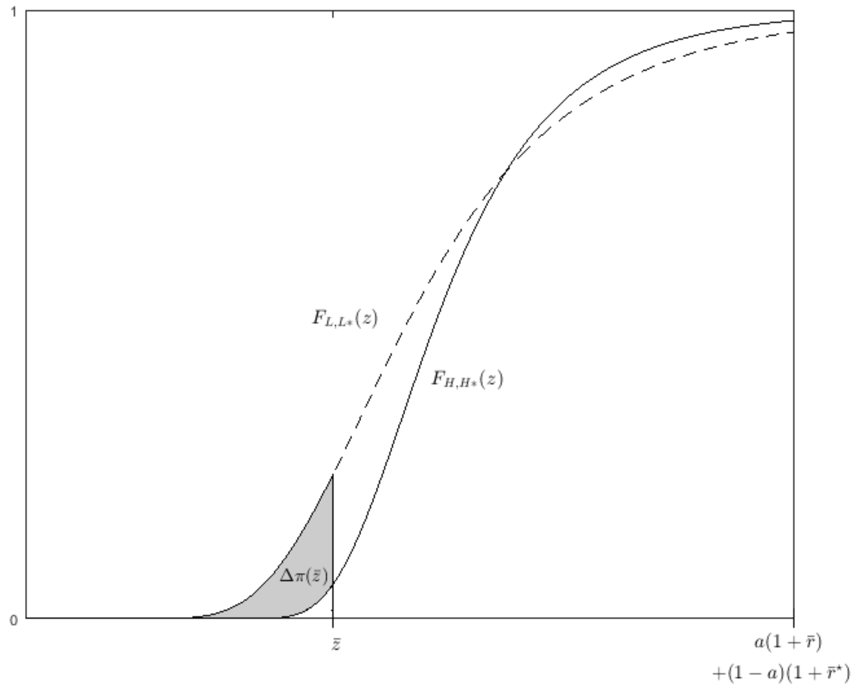


Figure 4 – The incentive compatibility constraint from the bank expected payoff: a unique solution $\bar{z}$. This chart plots the distribution functions $F_{H,H^{\star}}$ and $F_{L,L^{\star}}$ for ξ= 0.1, θ=$\theta^{\star}$=0.5, σ= 0.1, k= 0.05, and m= 1.4. The dark line indicates $F_{H,H^{\star}}$ and the dash line indicates $F_{L,L^{\star}}$.

3.3. Value at Risk

As in Adrian and Shin [2014], it is assumed here that $\xi = -1$ and $m \mapsto 1$.[13] Thus, the CDF of the mixture functions are of the form:

$$F_{H,H^{\star}}(z) = a\ exp\left\{\frac{z-\theta}{\sigma} - 1\right\} + (1-a)\ exp\left\{\frac{z-\theta^{\star}}{\sigma} - 1\right\}$$
$$F_{L,L^{\star}}(z) = a\ exp\left\{\frac{z-(\theta-k)}{\sigma} - 1\right\} + (1-a)\ exp\left\{\frac{z-(\theta^{\star}-k)}{\sigma} - 1\right\}$$

Hence $F_{L,L^{\star}} = e^{\frac{k}{\sigma}} F_{H,H^{\star}}$

[13] $\xi = -1$ implies that the $F_{H,H^{\star}}(z)$ distribution has an upper bound: the support of the distribution is $\left(-\infty\ ,\ -\sigma ln\left(a.exp\left\{-\left(\frac{\sigma+\theta}{\sigma}\right)\right\} + (1-a)exp\left\{-\left(\frac{\sigma+\theta^{\star}}{\sigma}\right)\right\}\right)\right)$. As the VaR rule focuses on the left side of the distribution, this assumption is not a problem. $m \mapsto 1$ makes the volatility between the good and the bad asset comparable. It allows an approximation of a closed form solution.

These assumptions allow the rhs of IC to be simplified as follows[14]

$$\begin{aligned} r_H - r_L &= \Delta\pi(\bar{d} + \bar{d}^\star) \\ &= (e^{\frac{k}{\sigma}} - 1)\sigma F_{H,H^\star}(\bar{d} + \bar{d}^\star) \end{aligned} \tag{14}$$

Because $F_{H,H^\star}$ is the bank's probability of default when it invests in the good portfolio, we can extract the following VaR rule from equation (14):

$$\alpha(\bar{d} + \bar{d}^\star) = F_{H,H^\star}(\bar{d} + \bar{d}^\star) = \frac{r_H - r_L}{(e^{k/\sigma} - 1)\sigma} \tag{15}$$

As the rhs of (15) does not depend on θ or $\theta^\star$, the probability of default α is maintained at the same level for any state of nature and any level of diversification. Especially, equation (15) defines the VaR rule where the bank adjusts its total debt ratio at T=1 $(\bar{d} + \bar{d}^\star)$ in order to satisfy a constant α. Following the VaR rule then implies both a constant α and a constant $\Delta\pi$. Note that the VaR rule focuses on the tail of the distribution. If the tail is thickened by a change in the state of nature, the bank has to decrease its total debt ratio in order to maintain a constant α that only depends on k, σ and the spread $r_H - r_L$.

Proposition 1 *Currency diversification does not affect the VaR rule. The bank adjusts its balance sheet to the state of nature in both currency areas: $(\bar{d} + \bar{d}^\star)$ adjusts to θ and $\theta^\star$ in order to satisfy a constant α.*

Equation (15) is equivalent to:[15]

$$\alpha = aF_H + (1-a)F_{H^\star} = \frac{r_H - r_L}{(e^{k/\sigma} - 1)\sigma} \tag{16}$$

$$\alpha = exp\left\{\frac{(\bar{d} + \bar{d}^\star) - \theta}{\sigma} - 1\right\}\left[a + (1-a)exp\left\{\frac{\theta - \theta^\star}{\sigma}\right\}\right] = \frac{r_H - r_L}{(e^{k/\sigma} - 1)\sigma} \tag{17}$$

The VaR rule determines bank leverage in its adjustment to the states of nature. The adjustment of $(\bar{d} + \bar{d}^\star)$ to the state of nature is:

$$(\bar{d} + \bar{d}^\star) = \theta + \sigma + \sigma ln\left(\frac{r_H - r_L}{(e^{k/\sigma} - 1)\sigma}\right) - \sigma ln\left(a + (1-a)\ exp\left\{\frac{\theta - \theta^\star}{\sigma}\right\}\right) \tag{18}$$

[14]See the appendix.

[15]I use the following arrangement: $F_{H^\star} = F_H \,.\, exp\left\{\frac{\theta - \theta^\star}{\sigma}\right\}$

If $a = 1$, there is no currency diversification and $(\bar{d} + \bar{d}^{\star})$ depends on the domestic state of nature θ. In contrast, if $a = 0$, only the foreign state of nature $\theta^{\star}$ affects $(\bar{d} + \bar{d}^{\star})$. The procyclicality of the leverage is derived from the degree of total debt ratio adjustment with respect to changes in both states of nature.

4. Procyclical leverage with currency diversification

4.1. Economic states of nature and the exchange rate

In previous sections, we assume a fixed exchange rate regime. However, changes in exchange rate will affect the weight of assets in the bank's portfolio since $a = \frac{A}{A+SA^{\star}}$. Depending on the correlation between the exchange rate and asset returns, a flexible exchange rate will interact with other channels of leverage adjustments.

Past empirical studies on Uncovered Interest Rate Parity (UIP) including Froot and Thaler [1990], MacDonald and Taylor [1992], have concluded to the inverse relationship of what the UIP theory predicts.[16] By distinghishing between short and long maturities, Chinn and Meredith [2004] confirm previous results and UIP failure, especially for short term maturity: the domestic currency tends to appreciate when domestic interest rates exceed foreign interest rates. Providing a small macroeconomic model with a feedback mechanism between exchange rates, inflation, output and interest rates, Chinn and Meredith [2004] explain this short term relationship with monetary authority's behavior defined as "leaning against the wind". Extending the analysis to equity markets and exchange rate, Ehrmann et al. [2011] use a structural VAR model with daily data from 1988 to 2004. They show that euro area markets rise significantly following an appreciation of the euro. Studying the reverse causality, they show that the euro is also positively affected by shocks on short rates where a rise in euro area short rates leads to a euro appreciation.

Hypothesis 1 *The domestic currency appreciates when the domestic return rises with respect to the foreign one.*

Additionally, as θ and $\theta^{\star}$ are known for both periods T=$\{0, 1\}$, the exchange rate S does not change between T=0 and T=1. Starting from an initial situation T<0 where $\theta = \theta^{\star}$, a symmetric increase in θ and $\theta^{\star}$ in the two economies does not change the interest rate spread.

[16] Following the IUP theory, a depreciation of the domestic currency should follow an increase in domestic returns in order to make returns equal among portfolios.

The exchange rate stays at its initial value $S = 1$. Now, if the amplitude of the positive change in the state of nature is larger in the domestic economy, $\theta > \theta^{\star}$, the domestic asset offers higher return: domestic currency appreciates and S decreases below 1 over T=$\{0, 1\}$. Finally, an anti-symmetric change in the states of nature such that θ increases while $\theta^{\star}$ falls by the same amount adds to the depreciation of the foreign currency. S falls well-below unity at T=$\{0, 1\}$. The process of S relative to good portfolios is given by equation (19) where return depends on the state of nature of both economies and on a function of the shape parameter $H(\xi)$:

$$S = 1 + \frac{r_{H^{\star}} - r_H}{1 + r_H} \tag{19}$$

$$\begin{aligned} &\text{Where :} \\ r_{H^{\star}} &= \theta^{\star} + \sigma H(\xi) \\ r_H &= \theta + \sigma H(\xi) \\ \lim_{r_H \to \infty} S(r_H) &= 0 \quad , \quad \text{and} \quad S = 1 \ \leftrightarrow \ r_H = r_{H^{\star}} \end{aligned}$$

As θ and $\theta^{\star}$ are known for both periods, the exchange rate does not change between T=0 and T=1. Implicitly, we also assume that the bank does not change the composition of its portfolio, notwithstanding small changes in states of nature.[17] When the domestic currency appreciates, the converted value of the foreign asset declines, which leads to a larger share of domestic assets relative to total assets: a goes up at T=$\{0, 1\}$. Consequently, the changes in a and $(1 - a)$ only reflect the exchange rate effect on converted value, or what we call the valuation effect of currency diversification. This makes it possible to identify the impact of currency diversification on leverage.

Hypothesis 2 *Changes in a only reflect valuation effects due to variations in the exchange rate, that is* $\frac{da(S)}{dS} < 0$.

[17]Hau and Rey [2008] provide evidences of portfolio rebalancing bahavior focusing on institutional investors between 1998 and 2002. Banks are then excluded from the analysis. Furthermore, Hau and Rey [2008] use half-years data while Odean [1998], Liu and Strong [2008] justify the "buy and hold" strategy for short term horizon because of the transaction costs implied in rebalancing strategies. Following Liu and Strong [2008], a monthly rebalancing strategy is then unrealistic. Because our model focuses on short term horizon, such as monthly horizon, we assume that banks do not change the composition of their portfolio.

We can rewrite equation (18) where a is a function of S such that:

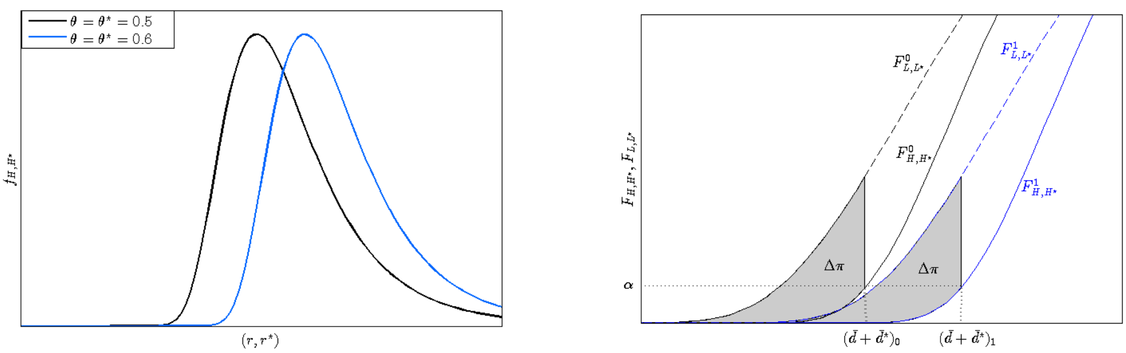
$$(\bar{d}+\bar{d}^{\star}) = \theta + \sigma + \sigma ln\left(\frac{r_H - r_L}{(e^{k/\sigma}-1)\sigma}\right) - \sigma ln\left(a(S) + (1-a(S))\ exp\left\{\frac{\theta-\theta^{\star}}{\sigma}\right\}\right) \quad (20)$$

With $\dfrac{da(S)}{dS} < 0$

4.2. A symmetric, positive change in both states of nature

If the two economies face a common positive or negative change in their state of nature, currency diversification does not affect the procyclicality of the leverage. Domestic and foreign assets still offer a similar return and the exchange rate is not affected. This specific positive change in states of nature is our benchmark for the comparative statics developed in sections 4.3 and 4.4.

Proposition 2 *Whatever the exchange-rate regime, currency diversification does not affect leverage procyclicality when changes in states of nature are symmetric, that is $\theta = \theta^{\star}$.*



(a) Distribution of portfolio shifts upward (PDF of the good portfolio returns)

(b) VaR rule with α and $\Delta\pi$ constant (CDF of portfolios returns)

Figure 5 – Global and positive change in states of nature: from $\theta = \theta^{\star} = 0.5$ to $\theta = \theta^{\star} = 0.6$. Black lines display initial PDF and CDF while blue lines illustrate new PDF and CDF. In b), less good portfolios are drawn with dashed lines. Distribution functions of both types of portfolio are defined for $a = 0.5$, $\xi = 0.1$, $\sigma = 0.1$, $k = 0.05$, and $m = 1.4$.

As illustrated in figure 5.a), the probability Density Functions (PDF) of the good portfolio total return shifts to the right because of the common positive change in states of nature. As total expected return goes up, the bank increases its total debt ratio $(\bar{d}+\bar{d}^{\star})$ to maintain

α and $\Delta\pi$ constant as in Adrian and Shin [2014]. The total debt ratio at the notional value goes from $(\bar{d}+\bar{d}^{\star})_0$ to $(\bar{d}+\bar{d}^{\star})_1$ in figure 5.b). The responsiveness of $(\bar{d}+\bar{d}^{\star})$ to changes in states of nature is not different from that of a single currency framework.[18]

4.3. An anti-symmetric change in states of nature

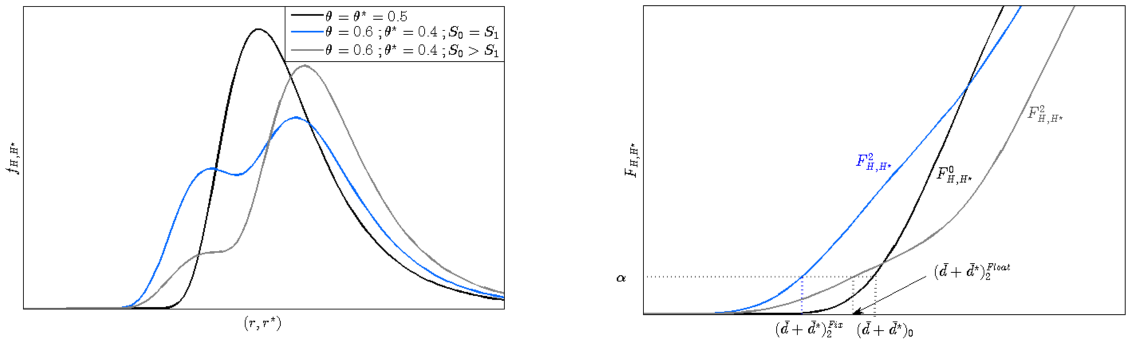
The anti-symmetric change in states of nature is either characterized by a positive change in the domestic's state of nature and an opposite change in the foreign economy's state of nature, or by a negative change in the domestic's state of nature and an opposite change in the foreign economy's state of nature. Depending on the anti-symmetric change in states of nature, we derive the following comparative statics result.

Proposition 3 *In both exchange rate regimes, and relative to the domestic state of nature, an anti-symmetric change in states of nature where $\theta > \theta^{\star}$ ($\theta < \theta^{\star}$) leads to counter-cyclical (less pro-cyclical) leverage when there is currency diversification.*

When the anti-symmetric change in states of nature is characterized by a positive change in the domestic's state of nature and an opposite change in the foreign economy's state of nature, the foreign asset's expected return decreases, implying a downward shift of its PDF. Inversely, the domestic asset's expected return increases, implying an upward shift of its PDF. As illustrated in blue in figure 6.a), the portfolio's PDF in a fixed exchange rate regime, which includes both assets' PDF, now includes two modes, one relative to each asset. Compared to the initial situation where $\theta = \theta^{\star}$, bank's portfolio is subject to increased risk of loss since the tail of the combined distribution is thickened. As the portfolio held by the bank becomes riskier, the bank has to deleverage to satisfy the VaR rule and the constant probability of default α.[19] Figure 6.b) shows this new adjustment where $(\bar{d}+\bar{d}^{\star})$ goes from $(\bar{d}+\bar{d}^{\star})_0$ to $(\bar{d}+\bar{d}^{\star})_2^{Fix}$. Compared to a portfolio only composed of domestic assets, leverage becomes counter-cyclical when diversification is introduced with fixed exchange rate regime.[20]

With a floating regime, the home currency appreciates. Therefore, the weight of domestic asset increases in bank's portfolio and the density relative to this mode goes up, as illustrated in grey in 6.a). As the risk of loss decreases relative to the fixed exchange rate regime, the

[18] Proof in the appendix 5.3.
[19] Satisfying the VaR rule with constant α is equivalent to that of a constant $\Delta\pi$; therefore, we choose to focus on the α condition relative to $F_{H,H^{\star}}$ in order to simplify graphic interpretations.
[20] Proof in the appendix 5.3.



(a) Distribution of portfolios
(PDF of the good portfolio returns)

(b) The VaR rule
(CDF of the good portfolio returns)

Figure 6 – Anti-symmetric change in states of nature: from $\theta = \theta^\star = 0.5$ to $\theta = 0.6$ and $\theta^\star = 0.4$. Black lines display initial PDF and CDF while blue lines and grey lines illustrate new PDF and CDF for fixed exchange rate ($a = 0.5$) and floating exchange rate ($a = 0.8$) respectively. Distribution functions of good portfolios are defined for ξ= 0.1 and σ= 0.1.

bank still deleverages to satisfy the VaR rule and α, but the adjustment is less brutal than with a fixed exchange rate regime. As illustrated in figure 6.b), total debt ratio moves to $(\bar{d} + \bar{d}^\star)_2^{Float}$.

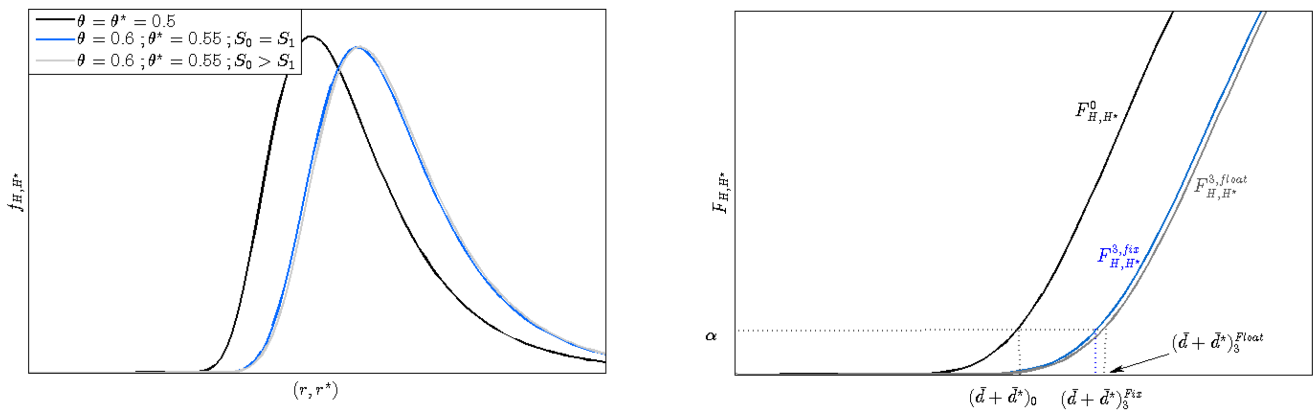
More generally, diversification associated with an anti-symmetric change in states of nature always leads to a decrease in leverage as the bank is subject to increased risk. Relative to domestic state of nature θ, leverage becomes counter-cyclical when $\theta > \theta^\star$, while it becomes less pro-cyclical when $\theta < \theta^\star$.[21] Because the exchange rate depends on the interest rate spread, a floating exchange rate implies an appreciation of the currency with higher returns. Thereby, it always increases in bank's portfolio the relative weight of the asset offering higher returns. Relative to domestic state of nature θ, leverage becomes less counter-cyclical compared to the fixed regime when $\theta > \theta^\star$, while it becomes even less pro-cyclical compared to the fixed regime when $\theta < \theta^\star$.

[21]When an anti-symmetric change is associated with $\theta < \theta^\star$, the decrease in leverage is less important with currency diversification than without currency diversification since the foreign asset improves total portfolio returns.

4.4. An asymmetric and positive change in states of nature

Lastly, we assume that the last change in states of nature is positive in both countries but the amplitude of the change is lower in the foreign economy. Thus, the return on assets differs and the distribution of the portfolio return flattens when the exchange rate is fixed. As illustrated in Figure 7, the blue PDF of the portfolio still shifts to the right but the density relative to the mode decreases. Consequently, the bank still increases $(\bar{d}+\bar{d}^{\star})$ but to a lesser extent than in the symmetric or global case. It reaches $(\bar{d}+\bar{d}^{\star})_3^{Fix}$ to keep α constant in 7.b). Leverage procyclicality is reduced.[22]

In a floating regime and relative to the home state of nature θ, procyclicality increases compared to the fixed regime. The asymmetric change in states of nature leads to the appreciation of the domestic currency. As the converted value of the foreign asset decreases, the domestic asset weight in the portfolio increases. In Figure 7.a) the grey PDF moves slightly to the right compared to the fixed exchange rate regime. Thus, bank leverage is more procyclical to satisfy the VaR rule and the constant α in figure 7.b) compared to the fixed regime. Total debt ratio rises from $(\bar{d}+\bar{d}^{\star})_0$ to $(\bar{d}+\bar{d}^{\star})_3^{Float}$.



(a) Distribution of portfolios
(PDF of the good portfolio returns)

(b) Leverage goes up to satisfy the VaR rule
(CDF of the good portfolio returns)

Figure 7 – Asymmetric and positive change in states of nature: from $\theta = \theta^{\star} = 0.5$ to $\theta = 0.6$ and $\theta^{\star} = 0.55$. Black lines display initial PDF and CDF while blue lines and grey lines illustrate new PDF and CDF for fixed exchange rate ($a = 0.5$) and floating exchange rate ($a = 0.6$) respectively. Distribution functions of good portfolios are defined for ξ= 0.1 and σ= 0.1.

[22] Proof in the appendix 5.3.

Regarding the two economic changes in states of nature mentioned above (anti-symmetric and asymmetric), a floating exchange rate regime always promotes the asset which offers a better return. As the bank follows a VaR rule, the floating exchange rate regime increases its capacity to raise funds compared to its debt capacity in a fixed exchange rate regime.

Proposition 4 *Compared to a fixed exchange rate regime, the introduction of a floating exchange rate increases the fund-raising capacity of the banks when changes in states of nature are anti-symmetric or asymmetric, that is* $\frac{d(\bar{d}+\bar{d}^{\star})}{dS} > 0$ *when* $\theta^{\star} > \theta$ *or* $\frac{d(\bar{d}+\bar{d}^{\star})}{dS} < 0$ *when* $\theta^{\star} < \theta$.

Depending on the relative state of nature across countries, currency diversification affects leverage procyclicality differently. Table 2 provides a summary of the results and extends the analysis to three other cases focusing on asymmetric changes in the state of nature. First, an asymmetric and positive change where $\theta < \theta^{\star}$ increases leverage procyclicality relative to the home state of nature: the rise in returns and the valuation effect magnify leverage adjustment compared to a portfolio only composed of a domestic asset. Second, an asymmetric and negative change where $\theta^{\star} < \theta$ increases leverage procyclicality compared to the home state of nature. The decrease in leverage is stronger than without diversification as the tail of the distribution is thickened by the foreign asset. However, $\theta^{\star} < \theta$ implies an appreciation of the domestic currency as the domestic asset offers higher returns than the foreign one. Therefore, the fund-raising capacity of the bank improves and the decrease in leverage is less pronounced with a floating than a fixed exchange rate regime. Finally, an asymmetric and negative change where $\theta < \theta^{\star}$ decreases leverage procyclicality relative to the home state of nature. Because $\theta < \theta^{\star}$, the foreign asset is less impacted by the negative change than the domestic asset. Therefore, diversification helps to mitigate total losses and the decrease in leverage is less strong than without diversification. Because the valuation effect promotes the foreign asset, leverage procyclicality is further weakened when the exchange rate is floating.[23]

More generally, diversification decreases leverage procyclicality when changes in states of nature are asymmetric and when the domestic economy is more affected than the foreign one. A floating exchange rate regime always increases the fund-raising capacity of the bank compared to a fixed exchange rate regime. Therefore, if a negative change in the states of nature is observed (either as an anti-symmetric change or as an asymmetric change), and

[23]See the appendix 5.4.

States of nature:	No diversification ($a = 1$)	Diversification ($a < 1$) Fixed regime	Floating regime
Symmetric change:			
$\theta = \theta^\star$	$+++$	$+++$	$+++$
Anti-symmetric change:			
With $\theta > \theta^\star$	$+++$	$--$	$-$
With $\theta < \theta^\star$	$---$	$--$	$-$
Asymmetric and positive change:			
With $\theta > \theta^\star$	+++	+	++
With $\theta < \theta^\star$	+	++	+++
Asymmetric and negative change:			
With $\theta^\star < \theta$	$-$	$---$	$--$
With $\theta < \theta^\star$	$---$	$--$	$-$

Table 2 – Leverage procyclicality, states of nature and exchange rate regimes: an increase in leverage is represented by a "$+$", while a decrease in leverage is given by a "$-$". The number of signs translates the magnitude of the adjustment.

if banks are diversified, the decrease in leverage is less pronounced with a floating exchange rate regime than with a fixed exchange rate regime.[24]

Conclusion

Global banks follow global strategies regarding the composition of their assets and liabilities, with marked regional diversification of balance sheets. According to the empirical literature, this diversification may have an impact on leverage procyclicality. However, no account has previously been taken of currency diversification, which affects the converted value of foreign assets in the balance sheet.

This paper offers the first theoretical model to introduce currency diversification in global banks' balance sheets. Based on Adrian and Shin [2014], the model micro-founds the VaR rule and confirms the active behavior of banks in response to economic fluctuations. Depending on the type of shock and on the exchange rate regime, we find that currency diversification affects leverage adjustment. When changes in states of nature are asymmetric and when the domestic economy is more affected than the foreign one, diversification decreases leverage procyclicality. When changes in states of nature are anti-symmetric with better conditions

[24] Proof in the appendix 5.5.

at the domestic level, leverage becomes counter-cyclical relative to domestic state of nature in both exchange rate regimes, while it becomes less procyclical relative to the domestic state of nature when conditions are better at the foreign level. Regarding the two economic changes in states of nature mentioned above (anti-symmetric and asymmetric), a floating exchange rate regime always promotes the asset which offers a better return. As the bank follows a VaR rule, the floating exchange rate regime increases its capacity to raise funds compared to its debt capacity in a fixed exchange rate regime: the floating exchange rate regime increases its risk-taking capacity.

Three policy implications can be derived from these results. First, as currency diversification is not neutral, regulators should monitor the degree of currency diversification in addition to geographic diversification. Second, regulators could encourage diversification with foreign assets if they offer a buffer against higher negative domestic shocks. Third, as a floating exchange rate regime helps to soften negative shocks when banks are diversified, regulators may want to promote floating exchange rate regime.

Finally, this paper develops a theoretical framework consistent with global banks' international involvement and advanced economies. Introducing currency diversification in banks' balance sheet and endogenous exchange rate provides a complete theoretical framework that can explain empirical heterogeneity in leverage procyclicality. First, it would be interesting to test these conclusions with micro data on banks' balance sheet. Second, it would be also interesting to extend this framework to a general equilibrium model. Thereby, the financial cycle would be connected to the real economic cycle and economic states of nature could be defined endogenously.

Appendix

5.1 Constant spreads:

As assets only differ in their location parameters, the spread between the good and the bad investment returns is equal for domestic as for foreign currency assets:

$$
\begin{aligned}
r_H - r_L &= \theta + \sigma H(\xi) - (\theta - k) - m\sigma H(\xi) \\
&= k - \sigma(m-1)H(\xi) \\
\text{And :} & \\
r_{H^\star} - r_{L^\star} &= \theta^\star + \sigma H(\xi) - (\theta^\star - k) - m\sigma H(\xi) \\
&= k - \sigma(m-1)H(\xi)
\end{aligned}
$$

Therefore:

$$
\begin{aligned}
& a(r_H - r_L) + (1-a)(r_{H^\star} - r_{L^\star}) \\
&= a.\left(\theta + \sigma H(\xi) - (\theta - k) - m\sigma H(\xi)\right) + (1-a)\left(\theta^\star + \sigma H(\xi) - (\theta^\star - k) - m\sigma H(\xi)\right) \\
&= a.\left(k - \sigma(m-1)H(\xi)\right) + (1-a)\left(k - \sigma(m-1)H(\xi)\right) \\
&= \left(k - \sigma(m-1)H(\xi)\right) \\
&= Cst
\end{aligned}
$$

5.2 IC development:

The simplifying assumptions give the following IC constraint:

$$
\begin{aligned}
(r_H - r_L) &= \Delta\pi(\bar{d} + \bar{d}^\star) \qquad \text{(IC)} \\
&= \int_0^{\bar{d}+\bar{d}^\star} F_{L,L^\star}\,\mathrm{d}z - \int_0^{\bar{d}+\bar{d}^\star} F_{H,H^\star}\,\mathrm{d}z \\
&= e^{\frac{k}{\sigma}} \int_0^{\bar{d}+\bar{d}^\star} F_{H,H^\star}\,\mathrm{d}z - \int_0^{\bar{d}+\bar{d}^\star} F_{H,H^\star}\,\mathrm{d}z \\
&= (e^{\frac{k}{\sigma}} - 1) \int_0^{\bar{d}+\bar{d}^\star} F_{H,H^\star}\,\mathrm{d}z \\
&= (e^{\frac{k}{\sigma}} - 1)\sigma F_{H,H^\star}(\bar{d} + \bar{d}^\star)
\end{aligned}
$$

5.3 Proofs of leverage adjustments:

The general expression of total debt ratio at the notional value is the following:

$$(\bar{d} + \bar{d}^{\star}) = \theta + \sigma + \sigma ln\left(\frac{r_H - r_L}{(e^{k/\sigma} - 1)\sigma}\right) - \sigma ln\left(a + (1-a)\ exp\left\{\frac{\theta - \theta^{\star}}{\sigma}\right\}\right)$$

Initially, the two economies are similar (e.g $\theta_0 = \theta_0^{\star}$) and total debt ratio is:

$$(\bar{d} + \bar{d}^{\star})_0 = \theta_0 + \sigma + \sigma ln\left(\frac{r_H - r_L}{(e^{k/\sigma} - 1)\sigma}\right)$$

When changes in economic state of nature are symmetric and positive, $\theta_1 = \theta_1^{\star}$ and total debt ratio is:

$$(\bar{d} + \bar{d}^{\star})_1 = \theta_1 + \sigma + \sigma ln\left(\frac{r_H - r_L}{(e^{k/\sigma} - 1)\sigma}\right)$$

Thus, leverage procyclicality is the following:

$$(\bar{d} + \bar{d}^{\star})_1 - (\bar{d} + \bar{d}^{\star})_0 = \theta_1 - \theta_0$$

Compared to Adrian and Shin [2014] leverage procyclicality is unchanged. When changes in economic state of nature are symmetric, whatever the exchange-rate regime, currency diversification does not affect leverage procyclicality.

Counter-cyclical leverage is observed when:

$$(\bar{d} + \bar{d}^{\star})_1 - (\bar{d} + \bar{d}^{\star})_0 < 0$$

$$\theta_1 - \theta_0 - \sigma ln\left(a + (1-a)\ exp\left\{\frac{\theta_1 - \theta_1^{\star})}{\sigma}\right\}\right) < 0$$

$$ln\left(\frac{1}{(1-a)}\right)(\theta_1^{\star} - \theta_0) < ln\left(\frac{a}{(1-a)}\right)(\theta_1^{\star} - \theta_1)$$

when the exchange-rate regime is fixed, $a = (1 - a)$ and the condition becomes:

$$\theta_1^\star < \theta_0$$

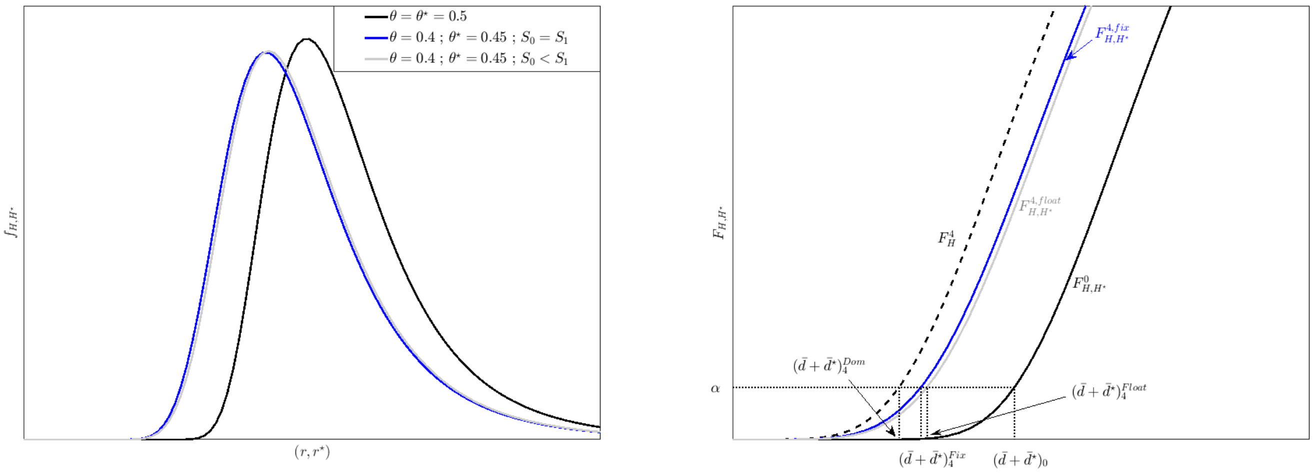
This condition is satisfied when changes in economic state of nature are anti-symmetric. Anti-symmetric changes in economic state of nature lead to counter-cyclical leverage.

More generally, leverage procyclicality is reduced when:

$$(\bar{d} + \bar{d}^\star)_1 - (\bar{d} + \bar{d}^\star)_0 < \theta_1 - \theta_0$$
$$a + (1 - a)exp\left\{\frac{\theta_1 - \theta_1^\star)}{\sigma}\right\} > 1$$
$$\theta_1 > \theta_1^\star$$

5.4 Asymmetric and negative change in the states of nature:

Figure 8 illustrates leverage adjustments when the change in states of nature is asymmetric and negative and when $\theta < \theta^\star$. As both states of nature decrease, the PDF distribution in blue shifts downward in figure 8.a) and tail of the distribution is thickened. Therefore, leverage decreases from $(\bar{d} + \bar{d}^\star)_0$ to $(\bar{d} + \bar{d}^\star)_4^{Fix}$ in figure 8.b). However, as the change in the foreign state of nature is less strong than the domestic one, the decrease in leverage is lower than the predicted decrease when the bank only holds domestic asset at $(\bar{d} + \bar{d}^\star)_4^{Dom}$. Compared to the domestic state of nature, leverage procyclicality then decreases with diversification. With a floating exchange rate regime, the PDF distribution still shifts downward compared to the initial situation in figure 8.a). However, as the foreign asset offers better return, foreign currency appreciates and the PDF distribution shifts upward compared to the fixed exchange rate regime in blue. As the fund raising capacity of bank increases, leverage improves from $(\bar{d} + \bar{d}^\star)_4^{Fix}$ to $(\bar{d} + \bar{d}^\star)_4^{Float}$ in figure 8.b). Therefore, a floating exchange rate regime decreases leverage procyclicality relative to the home state of nature.



(a) Distribution of portfolios (PDF of the good portfolio)

(b) Leverage goes down to satisfy the VaR rule (CDF of portfolios)

Figure .1 – **Asymmetric and negative change in states of nature:** from $\theta = \theta^\star = 0.5$ to $\theta = 0.4$ and $\theta^\star = 0.45$. Black lines display initial PDF and CDF while blue lines and grey lines illustrate new PDF and CDF for fixed exchange rate and floating exchange rate respectively. Dashed line in b) displays the CDF corresponding to the domestic asset when $\theta = 0.4$. Distribution functions of good portfolios are defined for ξ= 0.1 and σ= 0.1.

5.5 Exchange rate and fund raising capacity:

The total ratio of notional values of debt $(\bar{d} + \bar{d}^\star)$ depends on S, such that:

$$(\bar{d} + \bar{d}^\star) = \theta + \sigma + \sigma ln\left(\frac{r_H - r_L}{(e^{k/\sigma} - 1)\sigma}\right) - \sigma ln\left(a(S) + (1 - a(S))\ exp\left\{\frac{\theta - \theta^\star}{\sigma}\right\}\right)$$

The effect of a floating exchange rate regime on $(\bar{d} + \bar{d}^\star)$ compared to the fixed exchange rate regime is observed through the derivative of $(\bar{d} + \bar{d}^\star)$ relative to S when θ and $\theta^\star$ are fixed:

$$\frac{d(\bar{d} + \bar{d}^\star)}{dS} = -\sigma \frac{\frac{da(S)}{dS}\left(1 - exp\left\{\frac{\theta - \theta^\star}{\sigma}\right\}\right)}{a(S) + (1 - a(S))exp\left\{\frac{\theta - \theta^\star}{\sigma}\right\}}$$

When the exchange rate regime is floating, S does not affect $(\bar{d} + \bar{d}^\star)$ when:

$$\frac{d(\bar{d} + \bar{d}^\star)}{dS} = 0$$

Implying that: $\theta = \theta^{\star}$.
An appreciation of the foreign currency (i.e S increases) leads to an increase in $(\bar{d} + \bar{d}^{\star})$ when:

$$\frac{d(\bar{d} + \bar{d}^{\star})}{dS} > 0$$

Implying that:

$$\frac{da(S)}{dS}\left(1 - exp\left\{\frac{\theta - \theta^{\star}}{\sigma}\right\}\right) < 0$$

Because we assume that $\frac{da(S)}{dS} < 0$, then the condition becomes $\theta^{\star} > \theta$. An appreciation of the foreign currency leads to an increase in the fund raising capacity as long as $\theta^{\star} > \theta$ and $\frac{da(S)}{dS} < 0$.
Alternatively, an appreciation of the domestic currency (i.e S decreases) leads to an increase in $(\bar{d} + \bar{d}^{\star})$ when:

$$\frac{d(\bar{d} + \bar{d}^{\star})}{dS} < 0$$

Implying that:

$$\frac{da(S)}{dS}\left(1 - exp\left\{\frac{\theta - \theta^{\star}}{\sigma}\right\}\right) > 0$$

Because we assume that $\frac{da(S)}{dS} < 0$, then the condition becomes $\theta > \theta^{\star}$. An appreciation of the domestic currency leads to an increase in the bank's fund raising capacity as long as $\theta > \theta^{\star}$ and $\frac{da(S)}{dS} < 0$.

The conditions allowing an increase in fund raising capacities depend on the definitions of the model. The difference in the states of nature defines the exchange rate adjustment while $\frac{da(S)}{dS} < 0$ defines the portfolio adjustment relative to the exchange rate. In this framework, a floating exchange rate regime always increases the bank's fund raising capacity compared to a fixed exchange rate regime when $\theta \neq \theta^{\star}$.

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