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Hierarchy of Trade and Sequential Exporting

Vincent Boitier & Antoine Vatan

Highlights

- We link the hierarchy of trade in terms of destinations to the age of firms.
- We find that firms tend to Melitz predictions with age.
- We develop a simple model of export choice based on sequential exporting.
- This model explains i) why homogeneous firms may export differently and ii) the convergence towards Melitz' prediction.

Abstract

Export destination baskets of exporting firms do not seem to follow the hierarchy predicted by heterogeneous firms models. Existing literature reconciles theory and data by incorporating additional components of heterogeneity. We first show that we do not need these components to bridge the gap between data and theory. We highlight a strong correlation between the respect of hierarchy and the age of the firm. Without taking into account the age of firms, what we observe in the data is not a drawback but a snapshot of the convergence toward theory's prediction. Second, we develop a simple dynamic model that features this convergence. Our theoretical argument is based upon sequential exporting (choosing one destination at a time) and upon a baseline trade-off between attractiveness and competition.

Keywords

Export Choice, Dispersion in Strategies, Experience as Exporter, Sequential Exporting.

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RESEARCH AND EXPERTISE ON THE WORLD ECONOMY

Hierarchy of Trade and Sequential Exporting

Vincent Boitier & Antoine Vatan

1. Introduction

A key feature of Melitz (2003) and models à la Melitz with asymmetric countries (i.e., Chaney (2008), etc) is that they predict a hierarchy of trade in the sense that any firm selling to the $k+1$st most popular destination necessarily sells to the kth most popular destination as well. Moreover, as judiciously noted by Chaney (2014), firms that export to the kth most attractive market have a productivity between the productivity threshold for exporting to k and $k+1$. This means that firms with high productivity serve more difficult markets than firms with low productivity. This is the standard self-selection effect of firms into export markets. This fact is explained by the existence of increasing costs that prevent firms with insufficient productivity from entering difficult markets.

However, in a recent work, Eaton et al. (2011) show that this is not fully consistent with the data. While it is true that the average productivity of firms is shown to be increasing with the costs to enter the destination market (see, among others, Bernard and Jensen (1999), Bernard et al. (2007), Mayer and Ottaviano (2008), Berthou and Fontagné (2008) and Manova and Zhang (2009)), firms do not enter markets according to a hierarchy.

The present article aims to reassess the empirical and theoretical tenets of this hierarchy of trade. In particular, we establish a link between the respect of hierarchy by firms and their age as exporter. Empirically, we bridge the gap between Eaton et al. (2011) empirical findings and Melitz/Chaney prediction by highlighting the role of firms' dynamics and the role of sequential exporting.

To do so, we exploit French customs data over the period 1994-2008. We restrict our empirical analysis to the study of how French manufacturing firms enter different foreign markets. In line with Eaton et al. (2011), we find, on average, little support for the existence of a hierarchy of trade. We build an indicator that measures the exact deviation in the French firms' behavior from the Melitz/Chaney prediction. We again find that the deviation dramatically decreases with the age of firms as exporters. For new born exporters, the observed deviation is 73.35 %, whereas the latter is 29.39 % for firms that export since fourteen years.

Our empirical regularities suggest that the absence of a hierarchy of trade is a short-run phenomenon. In the long run (or, at least, with age), firms tend to follow a hierarchy of trade.

Theoretically, natural explanations could relied on experience of firms in the sense that firms acquired knowledge about markets (and/or analogically about their export costs) as they become older. Nevertheless, after doing our best to control for these stories, we still observe such a convergence. Our investigations suggest that this phenomenom is linked to sequential exporting defined as the fact that firms do not choose all their long-run destinations in one, but expand their destinations portfolio time after time.
As a result, given that the Melitz/Chaney model makes predictions concerning long-run economies, it does not account for these short-run characteristics. Thus, we argue that what we observe in the data is not a failure of this model. On the contrary, we reckon that there is an unambiguous convergence toward the Melitz/Chaney prediction. Our stylized fact could be simply summed up as follows: firms do not follow an exact hierarchy but converges to it with age. This statement has a theoretical corollary. When we observe two firms belonging to a same class of productivity, they diverge in their export destinations portfolio (whereas exact hierarchy would imply the same portfolio) but they tend to share the same as time runs. We provide a theoretical explanation based on sequential exporting to this latter corollary.

We develop a simple theoretical approach ti show that the baseline trade-off between accessibility and competition, that is present in any model with monopolistic competition and sequential exporting, is sufficient to generate the findings highlighted previously.

The static version of our model first expalins why strictly homogeneous firms may export differently. We deal with a fixed mass of homogeneous firms playing the following two-stage game. They choose a unique destination to export to, and once the destination is set, they monopolistically compete with other exporters for this destination.[1] Furthermore, the fact that the mass of firms is fixed implies that the free entry condition does not apply, and therefore, firms make profits. In that context, we study the behavior of the economy in the short run and show that firms face a trade-off. This trade-off is summarized by the fact that, naturally, profits decrease in relation to the density of competitors (i.e., fear of competition) and in relation to the distance to the destination due to transport costs (i.e., accessibility). Hence, firms have an incentive to export to the most attractive economies to avoid paying high transport costs. However, they anticipate that these destinations are precisely those to which a high number of competitors will export. As a consequence, some firms are encouraged to export to less attractive countries to escape competition. Based on this effect, equilibrium is a situation in which a non-degenerated distribution of firms with respect to their destination choices balances the two opposite forces summarized above.

Motivated by empirical evidence, we add periods to our baseline model. In our dynamic frame-

[1] This destination choice process finds empirical support. Notably, it has been documented that a substantial share of new exporters enters just one destination. For example, Albornoz et al. (2012) report that, among new Argentine manufacturing exporters between 2002 and 2007, 79 % of them serve a unique foreign market, whereas only 6 % of them enter more than three countries. We provide similar proportions with our data in the following section.

work, each firm chooses one destination per period (i.e., sequential exporting).[2] Unlike what is implied in long-run models with free entry, firms do not choose all of their destinations simultaneously, but rather one at a time. We demonstrate that our model tends to yield the same predictions as the Melitz/Chaney model. Over time, firms tend to export to the same ordered basket of destinations. This feature of the model is a result of our escape-competition effect. Therefore, our argument is that what we observe in the data is not a drawback but a snapshot of the convergence toward the Melitz/Chaney prediction.

This paper contributes to the literature on hierarchy of trade. In the recent literature, two articles (Eaton et al. (2011) and Chaney (2014)) address the issue that we examine. In both papers, firms can serve non-ordered sets of foreign markets. The common feature of these two models is that they introduce a second dimension of heterogeneity between firms. In the Eaton et al. (2011) model, the second dimension of heterogeneity that is added is market- and firm-specific heterogeneity in entry costs and idiosyncratic demand shocks. The model adds these factors by incorporating the Arkolakis (2010) formulation of market access. This extra element explains why firms make different choices regarding their decision to enter a given market: only the firm with the highest market-specific component of demand and/or the lowest market-specific component of fixed costs enters the country. Chaney (2014) also integrates a second dimension of heterogeneity between firms. He argues that firms can meet trading partners in two ways. On the one hand, they can meet trading partners by direct search, which is modeled as a geographically biased random search, while on the other hand, once a firm has acquired trading contacts in foreign locations, it can develop a new network from these locations: firms differ in their ability to develop a network of consumers in a given market. This new ingredient triggers the heterogeneity of export choices across firms: firms export to those markets where they are able to develop their network. In contrast to these two contributions, we propose a new rationale for the absence of a hierarchy of trade (even if there is convergence) without the need to add another heterogeneous dimension between firms.

The remainder of the paper is organized as follows. Section 2 introduces the empirical evidence. Section 3 presents the export choice model. Section 4 provides the conclusions.

2. Stylized Facts

2.1. Data and Definitions

We use French Customs data over the period 1194-2008. This dataset is disaggregated at the firm level (i.e. SIREN codes) and at the product level (NC8). Therefore, an observation in

[2]This hypothesis is empirically motivated by Defever et al. (2015) and Albornoz et al. (2012), who focus on the dynamics of exports. Using Chinese and Argentinean data, the latter shows that the export choices of firms feature a sequential entry into foreign markets.

the dataset is SIREN code, a NC8 code, a year and an export destination. However, the NC8 nomenclature is not stable over time and we do not think that such a disggregation level is required for ou purpose. For that reasons, we decide to use HS2 disaggregation level.
Consistently with standard theory, we consider single product firms. Thus, we call a "firm" a couple SIREN-HS2 code. Starting from this definition, we follow Eaton et al. (2011) in first steps of our analysis. Namely, we measure the attractiveness of a given country by the number of firms that export in this country. We borrow the definition of hierarchy of trade to Eaton et al. (2011) as well. We consider that a firm follows hierarchy if, given that she exports in the kth, she sells in $k-i$th, $i \in [1; k-1]$ markets.
First, we determine the eight most popular destinations to which French manufacturing products are exported. Table 1 displays these destinations for 2008 as well as the associated number of exporters.[3]

Destinations	Nb. of French Exporters
Belgium (BE)	23,793
Switzerland (CH)	9,490
Germany (DE)	6,700
Spain (ES)	4,050
Austria (AT)	3471
Canada (CA)	2590
United Kingdom (UK)	2,475
United States (US)	2,330
Any destination	55,140

Table 1 – Most Popular Destinations

As eaton et al. (2011), we find that only a small proportion of firms follow an exact hierarchy. In our sample, it concerns nearly 17% ($\frac{8943}{55140}$). It is even lower than Eaton et al. (2011). We do not aim at explaining this difference, but some explanations are natural candidates among which the crisis might play an important role. Nevertheless, what matters for our purpose is not the total proportion of firms that follow an exact hierarchy but the relation between this proportion and the age of firms as exporters. By the "age as exporter", we simply denote the number of years since the first apparition of the firm in our sample. We simply consider that a firm is a new exporter (age=1) at time t if she exports in t but not in $t-n$ ($n \in [1, t-1]$).

[3]Note that Belgium includes Luxembourg.

2.2. Hierarchy of Trade and Age as Exporter

With these definitions, we investigate the link between the age of firms and the hierarchy of trade.We first build 4 cohorts of firms: 1st year as exporters in 2008, 5th year as exporters in 2008, 10th year as exporters in 2008 and more than 14 years as exporters in 2008. To clarify, the cohort 10th year as exporters pools the firms that were 1st year exporters in 1999 and are still surviving and exporting 10 years thereafter. Table 2 displays the number of exporters according to the top eight destinations and the previously defined age groups. It is striking that destination choice is age-dependent. Surprisingly, for firms with 1 year of age, the most popular destination is Switzerland, then Belgium and then the United States. By contrast, firms that experience a long period of exporting share the same ranking as that shown in Table 1.

Destinations	1^{st} year	5^{th} year	10^{th} year	more that 14 years
Belgium (BE)	856	532	371	2040
Switzerland (CH)	1232	308	195	812
Germany (DE)	474	220	161	727
Spain (ES)	358	161	132	346
Austria (AT)	35	43	46	277
Canada (CA)	197	69	49	232
United Kingdom (UK)	243	109	68	260
United States (US)	568	148	81	159

Table 2 – French Firms Exporting to the Eight Most Popular Destinations and Age as an Exporter

This suggests that the probability to follow hierarchy increases with age. In 2008, firms that follow an exact hierarchy (i.e. exporting in all $k-n$ destinations when exporting in the kth) represent 8.6% of new exporters, 15.7% of 5 years exporters, 18.8% of 10 years exporters and 26.2% of exporters of at least 14 years.
This is the relationship we want to establish the most unambiguously as possible. To do so, in what follows, we study the "deviation" from the Melitz framework in terms of hierarchy.

2.3. Deviation from Melitz/Chaney and Age as Exporter

The results in the previous section do not give a complete view of the data. This is because computing the proportion of French exporters that adhere to a hierarchy of trade does not thoroughly measure the importance of the deviations from the Melitz prediction as firms that do not follow a hierarchy are not taking into account. Consequently, we consider the impact of these deviations from the benchmark by creating a consistence measure, and quantify how

these deviations evolve with experience. For each firm, we identify the least attractive country to which she exports in 2008 (say the $k-th$) and we look at all countries she serves. Each country with a rank $k-n, n \in [1, k-1]$ which is not served counts as an error. For this firm, the potential number of errors is $k-1$ (i.e. exporting only in k). We then compute the proportion of errors as the number of errors made upon the potential number of errors. This indicator goes from zero (the firm exports in every $k-n$) to one (she exports only in k).
We compute this indicator at two levels. First at the French Exporters level (i.e. as in previous section) and at the sector level (i.e. one ranking by HS2 code). We then compute the mean of these proportions by group of age. The following figure reports these means and displays a clear decreasing relationship with the age for both the ranking of the total sample and the ranking established at the HS2 level.

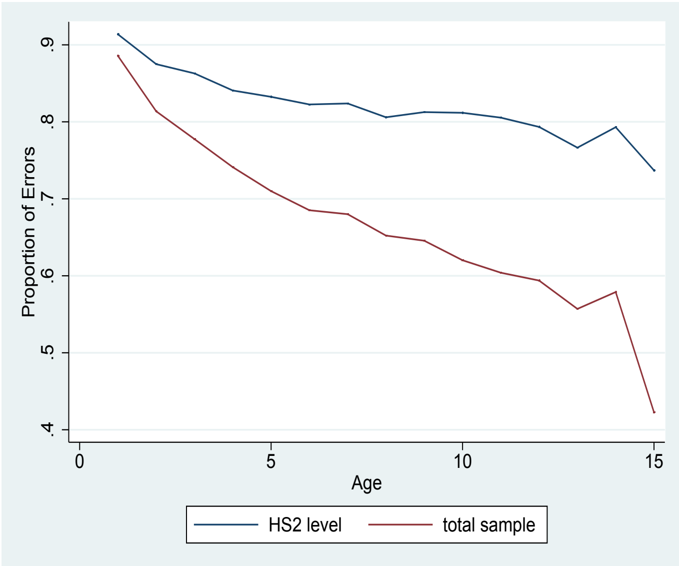

Figure 1 – Proportion of Errors and Age as Exporter

All these statistics suggest the older a firm, the closer her export destinations to the Melitz prediction. This would mean that what Eaton et al. (2011) observe with no distinction of age is not the complete story. We would rather suggest that what they observe in the data is not a failure of the Melitz prediction but a snapshot of the convergence towards Melitz (which is a long run model).

2.4. Deviation from Melitz/Chaney and Sequential Exporting

At this point, lot of things could be hidden beyond the "age" of firm. The reader may for instance naturally think to a learning process. We do not aim at proving that such phenomena do not exist. We only want to introduce our model where this convergence is at play but simply by a sequential exporting mechanism. To do so, we run few regressions linking the proportion of errors made by a firms to her age trying to control for alternative mechanisms. We compute the proportion of errors according to the ranking of countries defined year by year (even if this ranking is very stable over time) at the HS2 level. We drop from our sample all firms that appear in the first year of our data. We do this because we do not know when these firms export for the first time (it could be in 1994 or in 1979 as well for instance). This would bias our results as we would affect the same experience to these firms. We did not drop these firms in the descriptive statistics analysis because it was a pure cross section analysis in 2008. Here, we aim at tracking firms over time.
We actually run two regressions with two different fixed effects (FE) structure.[4] As we do not have many informations in the customs data, we think that it is the best we can do. The empirical strategy is simple. We regress the proportion of errors (p_{sht}) made by a firm at time t (a "firm" is a SIREN code s and a HS2 code h) on her age at t (age_{sht}) and on the log of her total export sales ($totex_{sht}$):

$$p_{sht} = \alpha + \beta \times age_{sht} + \gamma \times ln(totex_{sht}) + FE + \epsilon_{sht}. \tag{1}$$

A first regression is run with FEs at the firm (SIREN×HS2) level simultaneously with years FEs (columns 1 and 3 in table 3). The firm FE aims at controlling for all invariant characteristics of the firm while the log of total export sales aims at controlling for variant characteristics (especially productivity shocks). It could be argued that this latter proxy is not satisfactory. We then run the same regression with FEs at the SIREN×year and HS2 levels (columns 2 and 4 in the table 3). This is not completely sufficient either as a "firm" is not SIREN code, but a group SIREN-HS2. Nevertheless, keeping the log of total exports at the firm level (SIREN-HS2) in addition to this FEs sructure hopefully controls for most of the variation of productivity.

Signs of coefficients are the ones we expected. These regressions suggest that, after having done our best to control for hidden stories, firms tend to adhere to the Melitz hierarchy as they become older. We do not aim at proving that mechanisms such as learning by doing or uncertainty do not exist. We only try to shed light on the fact that this convergence seems

[4] In all regressions, errors are clustered at the SIREN level.

	(1)	(2)	(3)	(4)
	p_{sht}	p_{sht}	p_{sht}	p_{sht}
age_{sht}	-0.0116***	-0.0494***	-0.0108***	-0.0425***
	(0.00)	(0.00)	(0.00)	(0.00)
$ln(totex_{sht})$			-0.0260***	-0.0350***
			(0.00)	(0.00)
FEs	SIREN×HS2 + Year	SIREN×Year + HS2	SIREN×HS2 + Year	SIREN×Year + HS2
N	414286	241275	414201	241192
r2	0.67	0.66	0.68	0.68

Standard errors in parentheses

* p<0.05, ** p<0.01, *** p<0.001

Table 3 – Export Errors and Age

to be at play even after controling for these explanations. Moreover, notice that the learning-by-doing story in a context of perfect information would not display such convergence. In this case, firms would extend their basket of destinations time after time but they would follow exact hierarchy at each period. Their productivity would increase at each period which would allow them to choose additional destinations. But in a standard Melitz framework incorporating this learning-by-doing component, predictions on hierarchy would be the same. Survival firms would only climb the productivity ladder with experience but their export destinations' basket would be the one predicted by Melitz (2003) for their productivity class at each period. Such a convergence would appear in a context of imperfect information. This is the case for instance when firms discover their specific demand once they have exported. This story certainly exists. Nevertheless, if it was the sole mechanism at play, we would find convergence towards Melitz whether the firm expands their destinations' basket or not. For instance, if at $t = 1$, a new born firm exports in the first and in the third country, we find 0.5 proportion of errors. If at $t = 2$ she discovers that she made a mistake by exporting in the third country and now exports only in the first, we then find zero errors. The firms has reduced her number of destinations and has converged towards Melitz. We would find zero errors if she stops to export to the third and then exports to the second country (the number of destinations does not change) and if she exports to the three countries as well (the number of destinations increases). Therefore, after controlling for single firm specific components, we would find same convergence towards Melitz whether the firm raises her number of destinations or not. This is not what we have in the data.
In the table 4, we run the same regression as before but we split our total sample into two subsamples. The first sample contains firms for which the number of destinations at t is lower or equal to the number of destinations at $t-1$ (column 1). On the contrary, the second sample contains firms that have increased their number of destinations (column 2).

	(1)	(2)
	p_{sht}	p_{sht}
age_{sht}	0.0094***	-0.0120***
	(0.00)	(0.00)
$ln(totex_{sht})$	-0.0253***	-0.0041***
	(0.00)	(0.00)
FEs	SIREN×HS2 + Year	SIREN×HS2 + Year
N	263700	51065
r2	0.70	0.70

Standard errors in parentheses

* p<0.05, ** p<0.01, *** p<0.001

Table 4 – Export Errors and Destinations' Portfolio Expansion

Results suggest that when a given firm chooses additional destinations, she converges towards Melitz while it does not seem to be the case if not. If the only phenomenom behind the correlation between age and convergence towards Melitz was learning-by-doing and/or some uncertainty in the firm's demand (or equivalently in the cost structure), signs would be the same in the two columns of the table 4. In this table we do not use the (SIREN×Year+HS2) FEs structure as this latter would be a cross sectional analysis. Signs would be negative in both columns but it would capture the fact that for a given SIREN, a given year, the "older" HS2 line is closer to Melitz hierarchy. As we split sample using the number of destinations in t compared to $t-1$, this would reflect the fact that the "older" HS2 line has accumulated more destinations previously (before $t-1$).
Results in table 4 could simply reflect the fact that when a firm has a larger portfolio of destinations, she makes less errors. In order to control for that, we add the number of markets to which a firm exports (n_{sht}) and a dummy variable equal to one if the firm has expanded her portfolio between $t-1$ and t ($\mathbb{D}_{\Delta n>0}$) to our regression (of course, we drop new born firms from the sample). Results are displayed in the table 5.

Obviously, the number of destinations is negatively correlated to the proportion of errors. Nevertheless, the negative sign associated to our dummy variable means, after controlling for the number of destinations and the age, firms lower their proportion of errors by selecting additional destinations between two periods.
Overall, our results suggest that this convergence has something to do with sequential exporting pricisely defined as the fact that firms do not select all their long run destination in one, but

	(1)	(2)
	p_{sht}	p_{sht}
age_{sht}	-0.0036*	-0.0309***
	(0.00)	(0.00)
$n_s ht$	-0.0324***	-0.0215***
	(0.00)	(0.00)
$\mathbb{D}_{\Delta n>0}$	-0.2293***	-0.2681***
	(0.00)	(0.00)
$ln(totex_{sht})$	-0.0074***	-0.0128***
	(0.00)	(0.00)
FEs	SIREN×HS2 + Year	SIREN×Year + HS2
N	263695	124323
r2	0.78	0.76

Standard errors in parentheses

* p<0.05, ** p<0.01, *** p<0.001

Table 5 – Export Errors, Number of Destinations and Portfolio Expansion

rather choose additional destinations time after time. By choosing destinations gradually, firms converge towards melitz.

In the following section, we present a simple model of monopolistic competition with sequential exporting providing an explanation to this stylized fact. The explanation we propose is not based on an ad hoc assumption plugged into a long run model. We aim at demonstrating that a simple game of sequential exporting added to a competition mechnism already present in monopolistic competition models displays this convergence. With this model, we provide an explanation on both the fact that firms do not follow an exact hierarchy and converge towards this hierarchy.

3. Baseline Model

Motivated by the empirical regularities established in Section 2, we offer the simplest model we can think of that explains the findings highlighted previously. In other words, we provide a new export choice model that features firms' dynamics and explains among homogeneous firms:

1. Why firms do not obey a hierarchy of trade when they are young exporters.
2. Why firms converge toward the Melitz/Chaney prediction over time when considering sequential exporting

3.1. Environment

Let us consider a world composed of a home country and a continuum of foreign countries distributed on a Hotelling line $\mathcal{X} = [0, X]$. Foreign countries are ranked with respect to their distance $x \in \mathcal{X}$ to the home country. There are two goods in this world: one homogeneous good produced under perfect competition in all countries and one horizontally differentiated good produced under monopolistic competition in all countries. We assume that in each foreign country there is a mass $\tilde{\mu}(x)$ of foreign firms selling $\tilde{q}(x)$ units of the differentiated good. Firms take it as given and we can interpret $\tilde{\mu}(x)\tilde{q}(x)$ as an *ex ante* measure of the competition degree in x.

3.1.1. Supply Side

In the home country, there is a unit mass of (*ex ante*) homogeneous firms. All of the firms produce a variety of the horizontally differentiated good at the same marginal cost $c > 0$ and at the same fixed cost $f > 0$. These firms also have the possibility to export to a single foreign country. To export to x, the firms must incur a transport cost $\tau x > 0$ for each unit of exported good knowing that, once the destination is selected, they will be engaged in a monopolistic competition.[5] Note that the model can be derived by assuming that each firm chooses an exogenous number $1 < n < \infty$ of destinations (i.e., simultaneous exporting). Moreover, we respectively denote $q_i(x)$ and $Q(x)$ as the quantities sold in country x for variety i and for all varieties available in country x (i.e., the total quantity exported to country x) so that:

$$Q(x) = \int_{I_x} q_i(x) di + \tilde{\mu}(x)\tilde{q}(x) \qquad (2)$$

with $I_x = \mu(x)$ being the set of available varieties in x or, equivalently, the density of firms exporting to x.

3.1.2. Demand Side

In each foreign country, the population size is normalized to one, and the demand side is summarized by a representative consumer with homogeneous preferences. The upper-tier utility function of this consumer is quasi-linear:

$$\mathcal{U}(z, q(x)) = z + U(q(x)) \qquad (3)$$

[5] τ could also capture the notion of multilateral trade resistance. Likewise, f could also capture the notion of fixed export cost.

with z being the consumption of the Hicksian composite good produced under perfect competition and used as the numeraire. We define $q(x)$ as the vector of consumption of varieties of the horizontally differentiated good such that $q(x) = (q_i(x))_{i=0}^{I_x}$. The lower-tier utility of consuming the differentiated good is given by a quadratic utility function à la Melitz and Ottaviano (2008):

$$U(q(x)) = \int_{I_x} aq_i(x) - \frac{b}{2}q_i(x)^2 di - \frac{\gamma}{2}\left[\int_{I_x} q_i(x)di\right]^2 . \tag{4}$$

Parameter a represents the consumer's intrinsic evaluation of the differentiated good. It also stands as a measure of the quality of the good (see Di Comite et al. (2014)). It is the same for all varieties, which implies that firms sell a good that is only horizontally differentiated. Parameter b reflects substitutability between varieties. It is a constant and is equal for each variety, which means that no variety is more or less substitutable than another. Parameter γ captures the demand linkage between varieties. A higher γ indicates that varieties are less differentiated, and the marginal utility of consuming a unit of variety i decreases more rapidly with the consumption of any variety $j \neq i$. We suppose that a, b, γ are positive and the same in all countries. As a result, the demand function addressed to each single firm will be strictly the same in each country. Finally, the budget constraint of the representative consumer is:

$$z + \int_{I_x} p_i(x)q_i(x)di \leq y \tag{5}$$

with y as the exogenous income of consumers. In this environment, the marginal utility of income can be normalized to one (see Spence (1976) and Neary (2016) for further explanations), and each firm faces the same downward-sloping demand function in each x:

$$p_i(x) = a - bq_i(x) - \gamma Q(x). \tag{6}$$

Firms take $Q(x)$ as given: they do not anticipate their impact on aggregates. In this sense, there is no strategic interaction, and firms monopolistically compete with other firms exporting to the same destination.

3.2. Static Setting

Within this simple framework, domestic firms play the following two-step game:

1. They choose a single country to export to.
2. They (monopolistically) compete in quantity in country x with all firms selling in this location.

The first assumption refers to sequential exporting. While this assumption is key for our results, it is supported by data. Between 1994 and 2008, 86.3% of new born firms exported in only one destination while 8.4% exported in two destinations. Therefore, nearly 95% of new exporters has chosen less than three destinations. Now, isolating firms that extend their destinations portfolio, we observe that 59% do it by implementing one destination at a time. 21% choose two supplmentary destinations and 9% three supplementary destinations. Overall, the mean of supplementary destinations year by year is 1.9. Thus, assuming sequential exporting with the possibility for firms to choose one destination at a time is not far from the reality.

So, the entire program of a firm is given by:

$$\max_{x,q_i(x)} \pi(x, q_i(x)) = \max_{x,q_i(x)} \{p_i(x)q_i(x) - (c + \tau x)q_i(x) - f\} \tag{7}$$

with π as the profit of firms. The model is solved by backward induction. Stage 2 determines an equilibrium quantity sold by firms in location x denoted by $q^*(x)$. Stage 1 pins down an equilibrium distribution of firms in destination x or, comparably, an equilibrium number of varieties exported to foreign country x denoted by $\mu^*(x) = I_x^*$.

3.2.1. Stage 2

Given that a firm exports to x, Stage 2 solves the following program:

$$\max_{q_i(x)} \{[a - bq_i(x) - \gamma Q(x)]\, q_i(x) - (c + \tau x)q_i(x) - f\}\,. \tag{8}$$

It yields the following best response function:

$$q_i^*(x) = \frac{a - c - \tau x - \gamma Q(x)}{2b}. \tag{9}$$

As all domestic firms are homogeneous in terms of their marginal cost, in addition to the fact that best response functions are linear in $Q(x)$, all firms that export to the same destination choose the same quantity. Let this quantity be $q(x)$. Hence, the total quantity sold in each country is simply $Q(x) = \mu(x)q(x) + \tilde{\mu}(x)\tilde{q}(x)$, and the equilibrium quantity produced by a firm exporting to x becomes:

$$q^*(x) = \frac{a - c - \gamma\tilde{\mu}(x)\tilde{q}(x) - \tau x}{2b + \gamma\mu(x)}. \tag{10}$$

With this quantity, the equilibrium profit made by a firm that exports to x is simply:

$$\pi(x, q^*(x)) = bq^*(x)^2 - f. \tag{11}$$

Because the equilibrium profit is strictly increasing in quantities, it is a decreasing function of both distance (at a given transport cost level) and the *ex ante* competition degree captured by $\tilde{\mu}(x)\tilde{q}(x)$. It is decreasing in the mass of firms exporting to x as well. The latter captures the endogenous part of the competition degree. Finally both $\mu(x)$ and $\tilde{\mu}(x)\tilde{q}(x)$ capture the *ex post* competition degree.

3.2.2. Stage 1

For a given equilibrium in stage 2, firms choose a destination so that:

$$\max_x \pi(x, q^*(x)) = \max_x \left\{ b \left(\frac{a - c - \tilde{\mu}(x)\tilde{q}(x) - \tau x}{2b + \gamma\mu(x)} \right)^2 - f \right\}. \quad (12)$$

Equation (12) emphasizes that firms face a trade-off. The trade-off is captured by the decreasing relationships between profits π and both x, the distance, and $\mu(x)$, the number of domestic competitors. Thus, it represents the desire to export to the most attractive countries and the desire to escape competition. The intuition is easy to grasp. As transport costs increase with distance, firms export to attractive countries. However, they also anticipate that more competitors will export to these countries. Therefore, to escape competition, some firms have an incentive to export to less attractive countries. This program also shows that firms select a destination for their exports in accordance with their preferences and the strategies of others, summarized by the density of competitors. In that particular case, the suitable equilibrium takes the following form:[6]

Definition 1 *A distribution of firms $\mu^* \in \mathcal{M}(\mathcal{X})$ is an equilibrium if, and only if:*[7]

$$\begin{cases} \pi(x, q^*(x)) = \pi(x, \mu^*(x)) \leq \pi^* & \textit{for almost every } x \in \mathcal{X} \\ \pi(x, q^*(x)) = \pi(x, \mu^*(x)) = \pi^* & \textit{for almost every } x \in \mathcal{X} \textit{ such that } \mu^*(x) > 0 \end{cases} \quad (13)$$

An equilibrium is a situation in which each homogeneous domestic firm receives the same total profit wherever it exports to because in such a configuration, unilateral deviations of strategies are impossible. Using (12) and Definition 1, we obtain the following proposition:

Proposition 1 *A unique distribution μ^* exists such that: $\forall x \in [0, \check{x}^*]$*

$$\mu^*(x) = \frac{a - c - \tilde{\mu}(x)\tilde{q}(x) - \tau x}{\gamma q^*} - \frac{2b}{\gamma} \quad (14)$$

with $\check{x}^*$ the threshold country above which no firms export:

$$\check{x}^* = \frac{a - c - \tilde{\mu}(x)\tilde{q}(x) - 2bq^*}{\tau} \quad (15)$$

with q^* a constant equal to:

$$q^* = \frac{4\left[a - c - \tilde{\mu}(x)\tilde{q}(x)\right)] b + 2\gamma\tau - \sqrt{16\left[a - c - \tilde{\mu}(x)\tilde{q}(x)\right] b\gamma\tau + 4\gamma^2\tau^2}}{8b^2} \quad (16)$$

and with π^* the equilibrium profit given by:

$$\pi^* = b(q^*)^2 - f \quad (17)$$

[6]See Cardaliaguet (2012) for further explanations regarding the form of the equilibrium.

[7]$\mathcal{M}(\mathcal{X})$ is the set of absolutely continuous Borel probability measures on $\mathcal{X}$ (absolutely continuous with respect to the Lebesgue measure).

The first striking result, sketched in Proposition 1, is that the firm's distribution is never degenerated. This result implies that homogeneous firms do export differently in the short run (i.e., they do not follow a hierarchy of trade in the short run). As outlined previously, this feature is due to our trade-off. Firms want to export to the most attractive countries to avoid paying high transport costs. However, at the same time, because they have the same profit function, they anticipate that these markets will be more coveted by their competitors. The competition will be tougher and will prompt some firms to deviate by exporting to less attractive destinations.

Unlike the results in Proposition 1, which seem to be cumbersome, the effect of each parameter on the firm's distribution is unambiguous (see Proposition 2). First, it is clear that the *ex ante* competition degree in x has a negative influence on $\mu(x)$. This been said, one can assume any form of $\tilde{Q}(x) = \tilde{\mu}(x)\tilde{q}(x)$ one can think of results would be the same in nature as long as doemstic firms take it as given. Therefore, for the sake of simplicity and for the rest of the analysis, we consider $\tilde{Q}(x)$ to be a constant.
Even if we deal with homogeneous firms as our first aim is to generate heterogeneity in strategies among homogeneous players, starting from that latter assumption, one can intuitively deduce the influence of a firm's productivity on the destination choice. We see that the slope of μ^* with respect to x is:[8]

$$\frac{\partial \mu^*(x)}{\partial x} = -\frac{\tau}{\gamma q^*} < 0 \tag{18}$$

This slope is steeper for a higher c because the equilibrium profit π^* is a decreasing function of c. Thus, for a higher productivity (i.e., lower c), the equilibrium profit is higher, which in turn implies a more slight influence of distance on trade. Moreover, plugging (16) into (15) gives:

$$\check{x}^* = \frac{-2\gamma\tau + \sqrt{16\left[a - c - \tilde{\mu}(x)\tilde{q}(x)\right] b\gamma\tau + 4\gamma^2\tau^2}}{4b\tau} \tag{19}$$

The threshold country $\check{x}^*$ is itself a decreasing function of c. Consequently, for a lower c, the distribution μ^* is flatter. This feature suggests that, on average, more productive firms export further, which is consistent with the data (see, for example, Bernard and Jensen (1999)). Another interesting result lies in parameter a. An increase in a has exactly the same impact as a decrease in c. Because this parameter can be interpreted as a measure of quality, our model predicts that firms producing high quality goods generate more profits and integrate into more distant markets than firms producing low-quality goods. This theoretical result goes along with recent empirical findings highlighted by Martin and Mayneris (2015) and Fontagné and Hatte (2013). Last, f, the fixed cost, does not determine firms' location decision, it only reduces firms' profit. Here, firms' self-selection only results from the trade-off emphasized previously between accessibility and competition. However, as in the basic trade theory, f prevents firms with insufficient productivity from exporting; only firms with $c < \overline{c}$ export with $\overline{c}$ being the productivity threshold.[9]

[8]Note that (18) shows that the extensive margin is decreasing with distance, which is consistent with the findings of Bernard et al. (2007).

[9]We obtain: $\pi^* = 0 \Leftrightarrow 4\left[a - \overline{c} - \tilde{\mu}(x)\tilde{q}(x)\right] b + 2\gamma\tau - \sqrt{16\left[a - \overline{c} - \tilde{\mu}(x)\tilde{q}(x)\right] b\gamma\tau + 4\gamma^2\tau^2} = 8b^{\frac{3}{2}}\sqrt{f}$.

Using (11), (16) and (17), it also results that:

$$q^*(x) = q^* \tag{20}$$

The equilibrium quantity sold by firms (i.e., the intensive margin) does not hinge on distance anymore, which reflects, once again, the fact that at equilibrium, the distance effect and the competition toughness compensate for each other. This result also means that our model can be viewed as a model of the extensive margin only. In so doing, we are in line with (among others) Chaney (2014). There are three reasons for leaving this outcome unchanged. First, in our empirical examination, our purpose has been to focus on the extensive margin of trade; we have deliberately omitted the study of the intensive margin. Second, we want to keep the setup simple. Embedding heterogeneity in the intensive margin of trade in the existing model can be cumbersome and may rule out the possibility of obtaining analytical and interpretable results. We leave this extension for future research. Third, this approach permits us to obtain a new, interesting prediction concerning firms' mark-up. Note that the price faced by firm i in each country x can be expressed as a function of the extensive margin $\mu^*(x)$:

$$p_i^*(x) = a - [b + \gamma\mu^*(x)]\, q^* \tag{21}$$

Because $\mu^*(x)$ is, as mentioned above, decreasing with distance, $\frac{p_i^*(x)-c}{c}$, the endogenous mark-up over the marginal cost is increasing with distance.This feature is consistent with the data. For instance, as proved by Bellone et al. (2016), mark-ups are negatively correlated to competition degree and positively to distance. In our model, this is not explained by the standard quality differentiation argument (Manova and Zang (2012 and Crozet et al. (2012)), but precisely because distance and competition degree are (endogenously) negatively correlated.

3.3. Dynamic Setting

By choosing one destination at a time, homogeneous firms are forced to have different strategies to escape competition, which is what we highlighted in the previous section by presenting a single-period version of the model. This section aims at adding several periods to stress the positive link between the age as exporters when considering sequential exporting and the fact that firms tend to follow a hierarchy of trade.

To achieve this goal, we add periods to our basic model so that, at every t, firm i faces the following downward-sloping demand function in each x:

$$p_{i,t}(x) = a - bq_{i,t}(x) - \gamma Q_t(x) \tag{22}$$

where $Q_t(x)$ is the number of firms that export to x at time t. Furthermore, as firms are myopic (i.e., they make decisions only according to their current utilities), the model resolution is unchanged. Stage 2 pins down an equilibrium quantity sold by firms in location x at time t denoted by $q_t^*(x)$:

$$q_t^*(x) = \frac{a - c - \tau x - \gamma Q_t(x)}{2b} \tag{23}$$

and Stage 1 determines an equilibrium density of firms at time t denoted by $\mu_t^*(x)$.

3.3.1. Sequential Exporting

In what follows, each firm is now allowed to choose a supplementary destination at each period as soon as profits are positive.[10] This approach naturally implies that, at each period $t > 1$, all firms select an additional destination. Indeed, one can demonstrate that it is always optimal for a firm to enter a new market at $t+1$. One can guess that the total profits at t are $t\pi_t^*$. If a firm did choose a supplementary destination at t, then its profits would be $(t-1)\pi_t^*$. This approach also implies that firms do not have incentives to exit markets to which they are already exporting. Therefore, they continue to export to the destinations chosen at $t-k$ with $k \in \{1, t-1\}$.

Moreover, at each period $t > 1$, we assume that a proportion ρ of exporters present at $t-1$ disappears, and a mass ρ of new firms enter and choose a destination.[11] Firms that survive keep producing and have the possibility of choosing an additional country to serve.

In that new case, the quantity sold in each x by each firm at t equals:[12]

$$Q_t(x) = \begin{cases} q_t^*(x)\mu_t(x) + \tilde{\mu}(x)\tilde{q}(x) \text{for } t = 1 \\ q_t^*(x)\left[\mu_t(x) + \sum_{k=1}^{t-1}(1-\rho)^{t-k}\mu_k^*(x)\right] + \tilde{\mu_t}(x)\tilde{q_t}(x) \quad \text{if } t > 1 \end{cases} \tag{24}$$

where $\sum_{k=1}^{t-1}(1-\rho)^{t-k}\mu_k^*(x)$ represents the number of exporters still surviving in t at destination x. Moreover, the equilibrium profit made by a firm that exports to x at t remains the same $\pi_t(x, q_t^*(x)) = bq_t^*(x)^2 - f$, but now, due to entries and exits, the firms differ according to their total profit $\Pi_t = k\pi_t(x, q_t^*(x))$, $\forall k \in \{1, t\}$. Therefore:

Definition 2 *A distribution $\mu_t^* \in \mathcal{M}(\mathcal{X})$ is an equilibrium if, and only if:*

$$\begin{cases} \pi_t(x, q_t^*(x)) = \pi_t(x, \mu_t^*(x)) \leq \pi_t^* & \text{for almost every } x \in \mathcal{X} \\ \pi_t(x, q_t^*(x)) = \pi_t(x, \mu_t^*(x)) = \pi_t^* & \text{for almost every } x \in \mathcal{X} \text{ such that } \mu_t^*(x) > 0 \end{cases} \tag{25}$$

Using Definition 2, we obtain:

Proposition 2 *For $t = 1$, a unique distribution μ^* is given by (14)-(17). For $t > 1$, a unique distribution μ^* exists such that: $x \in [0, \check{x}^*]$*

$$\mu^*(x) = \frac{a - c - \tilde{\mu_t}(x)\tilde{q_t}(x) - \tau x}{\gamma q^*} - \frac{2b\rho}{\gamma} \tag{26}$$

[10] One could consider the same framework using $1 < n < \infty$ destinations or a certain entry/exit rate of firms. One could also assume that only a proportion ψ of firms are allowed to choose a supplementary destination. In all cases, the nature of the results is the same, although these extensions make the formulas much more complex.
[11] Several extensions could be considered. For example, one could assume that the proportion of exporters present at $t-1$ in x that disappears is destination and time specific (i.e., $\rho_t(x)$).
[12] For simplicity, we suppose that $\tilde{\mu}(x)\tilde{q}(x)$ is constant over time.

with:

$$\breve{x}^* = \frac{a - c - \tilde{\mu}_t(x)\tilde{q}_t(x) - 2b\rho q^*}{\tau} \tag{27}$$

$$q^* = \frac{4\left[a - c - \tilde{\mu}_t(x)\tilde{q}_t(x)\right] b\rho + 2\gamma\tau - \sqrt{16\left[a - c - \tilde{\mu}_t(x)\tilde{q}_t(x)\right] b\rho\gamma\tau + 4\gamma^2\tau^2}}{8b^2\rho^2} \tag{28}$$

and

$$\pi_t^* = b\left(\frac{q_1^*}{\sum_{k=0}^{t-2}(1-\rho)^k q_1^* + (1-\rho)^t}\right)^2 - f \tag{29}$$

where π_t^* decreases with respect to t and q_1^* given by (16).

We also plot two distributions in Figure 4. The dotted lined represents the initial density distributions at time $t = 1$. We label this distribution μ_0^*. The solid line depicts the same distribution but at time $t > 1$. The latter is denoted by μ_t^*. Coupling Proposition 3 and Figure 4, several issues emerge.

First, we obviously find the same expression as that found in Section 3.2 for the spatial distribution of firms at $t = 1$. Up to parameter ρ, the equilibrium distribution at time $t > 1$ is identical to that at time $t = 1$. This means that, at time $t > 1$, we obtain a flatter distribution. More surprising, the obtained distribution becomes constant over time: the threshold country increases between the first and the second period, and then it stays the same beginning in the second period. The phenomenon is the same for the trade gradient. Until the appearance of these differences, the predictions made in Section 3.2 are preserved in this dynamic setting. Notably, the effects of the parameters are the same: the firms' distribution is steeper when c increases and a decreases.

Second, adding periods is strictly equivalent to allowing surviving firms to choose another destination, competition is increasingly more difficult with time, and profits in a given destination decrease over time. This decrease in profits implies that there exists a unique period $t^* < \infty$ that leads to zero profits for all firms.[13] This feature of the model is explained by the entry of new firms and the fact that incumbent firms are engaged in sequential exporting. This intensifies the competition between foreign destinations and, at a given time period, this becomes unprofitable for a firm to continue entering the game.

Third, a corollary to the Melitz/Chaney model in such a framework would be that all firms with a given level of productivity should export to all countries between 0 and $\breve{x}^*$. This result is not exactly the case in our model. At any $t > 1$, the threshold country $\breve{x}$ is the same, and all surviving firms select a supplementary destination. Also note that $\mu^*(x)$ can be viewed as the

[13] More precisely, $\exists\ t^*$ such that: $t^* = \lfloor \tilde{t} \rfloor$ and $\Pi_{\tilde{t}}^* = 0$.

probability for a firm to export to x at t. Because $\mu^*(x)$ decreases with x, it implies that firms are more likely to enter new destinations near $x = 0$ than near $x = X$. Therefore, two firms observed at t are more likely to have similar baskets of destinations than if they were observed at $t' < t$. Hence, a prediction of our model is that firms tend to have more similar ordered strategies. Nevertheless, as stated previously, because π_t^* is decreasing over time, there exists a unique period t^* that leads to zero profits and, in the long run, firms export to a finite number of t^* destinations. As a consequence, the model never exactly converges toward the Melitz/Chaney prediction. It is still true that firms tend to have more similar baskets of destinations over time, but at t^* there still exists dispersion.

3.3.2. Extrapolation

To be a bit more illustrative, imagine we simulate our game, generate results and observe the latter at time t. What do we observe? First, a cross sectional analysis reveals that older firms make less mistakes and export to more destinations (remind that we are dealing with homogeneous firms). Second, a longitudinal analysis reveals that, as firms become older, they make less and less mistakes according to a hierarchy in export destinations. This is consistent with our (real) data analysis, and cannot be explained in a context of standard long-run monopolistic competition models of trade, except if we add *ad hoc* heterogeneity among firms. In a way, our model describes the convergence towards the long-run equilibrium of Melitz (2003) (which is never reached as firms die before). This dynamics of entry describes in fact how we converge towards free entry. Dealing with long-run equilibrium is actually the same as taking our model, assuming $\rho = 0$ and $t \to \infty$. Therefore, we claim that Melitz (2003) is not contradicted by data, it is a theoretical horizon which cannot be reached. But in pure abstraction, this horizon would be reached if world were stable over time, and firms infinitely lived. We are not claiming that a firm-specific fixed cost and demand structure (as in Eaton et al. (2011)) does not exist. It surely exists. We are claiming that it is not the full story, even if it is the most obvious and probably the one explaining the highest share of variance among a same class of productivity. We are claiming that it is realistic to think that between two strict identical firms, there are heterogeneous destinations. This can be explained by our model which in essence uses a traditional argument: firms want to escape competition and exporting is a way to do so.

4. Conclusions

The present paper tackles the issue of hierarchy of trade. Using Customs French data, we first document that the fact that French exporters follow a hierarchy of trade depends on export experience. This means that they strongly export to different markets when they are young exporters, while, conditional on survival and over time, they tend to export to the same ordered set of countries. In so doing, we reconcile Melitz (2003) and models à la Melitz augmented by asymmetric countries with Eaton et al. (2011). Second, we offer a new model of export choice that provides a rationale of this new stylized fact. Based on a simple trade-off that is present in any model of monopolistic competition with sequential exporting, we show how two identical firms can enter different markets. Our economic argument is that trade can be considered as a way to escape competition. In a multi-period setting, our model displays the dynamics emphasized by the empirical part, namely, that firms tend to have more similar strategies as they become more experienced exporters. However, though the model converges toward the Melitz/Chaney prediction, it never reaches it fully. Thus, we argue that what we observe in the data is not a failure but a convergence toward the Melitz/Chaney prediction.

Several extensions can be considered. In particular, we see, among others, three possible refinements. First, we have called for an additional extension: the inclusion of heterogeneity in the intensive margin of trade. Second, our article is qualitative in nature. It aims at highlighting a new mechanism of firms' self selection. As a consequence, future research should be directed toward quantitatively matching the empirical regularities that we have highlighted. Third, we derive new predictions concerning the relationship between mark-up and distance and gains from trade that may provide a direction for empirical works. We leave all of these issues for future studies.

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Appendix

6. Proofs

Proof 1 *π is continuous and $\frac{\partial \pi(x,q^*(x))}{\partial \mu(x)} < 0$. From Cardaliaguet (2012), this implies that a unique μ^* exists. Then, we determine $\mu^*(x)$ a density of firms such that:*

$$\begin{cases} \pi(x,q^*(x)) = b\left(\frac{a-c-\tilde{\mu}(x)\tilde{q}(x)-\tau x}{2b+\gamma\mu^*(x)}\right)^2 - f = \pi^* \in \mathbb{R}_+^* & \forall x \in \mathcal{X} \\ \int_{Supp(\mu^*)} \mu^*(x)dx = \int_0^{\breve{x}^*} \mu^*(x)dx = 1 \\ \mu^*(x) \geq 0 & \forall x \in \mathcal{X} \end{cases}$$

With the first condition of the system, we immediately find:

$$\mu^*(x) = \frac{a-c-\tilde{\mu}(x)\tilde{q}(x)-\tau x}{\gamma\sqrt{\frac{\pi^*+f}{b}}} - \frac{2b}{\gamma}$$

Note that the trade gradient is $tg^* = \frac{\partial \mu^*(x)}{\partial x} = -\frac{\tau}{\gamma\sqrt{\frac{\pi^*+f}{b}}}$ and $\mu^*(0) = \frac{a-c-\tilde{\mu}(0)\tilde{q}(0)-2b\sqrt{\frac{\pi^*+f}{b}}}{\gamma\sqrt{\frac{\pi^*+f}{b}}}$. Using the third condition of the system, the threshold country $\breve{x}^*$ is determined as $\mu^*(\breve{x}^*) = 0$, that is,

$$\breve{x}^* = \frac{a-c-\tilde{\mu}(x)\tilde{q}(x)-2b\sqrt{\frac{\pi^*+f}{b}}}{\tau}$$

If $0 < \breve{x}^* < X$ then integrating μ^* on its endogenous support $[0, \breve{x}^*]$ yields the following quadratic polynomial

$$4b^2(q^*)^2 - [4[a-c-\tilde{\mu}(x)\tilde{q}(x)]b + 2\gamma\tau]q^* + [a-c-\tilde{\mu}(x)\tilde{q}(x)]^2 = 0$$

with $q^* = \sqrt{\frac{\pi^*+f}{b}}$. The discriminant of this quadratic polynomial is:

$$\Delta = [4[a-c-\tilde{\mu}(x)\tilde{q}(x)]b + 2\gamma\tau]^2 - 16[a-c-\tilde{\mu}(x)\tilde{q}(x)]^2 b^2 = 16[a-c-\tilde{\mu}(x)\tilde{q}(x)]b\gamma\tau + 4\gamma^2\tau^2 > 0$$

Because $\Delta > 0$, it admits two solutions:

$$Sol_1 = \frac{4[a-c-\tilde{\mu}(x)\tilde{q}(x)]b + 2\gamma\tau - \sqrt{16[a-c-\tilde{\mu}(x)\tilde{q}(x)]b\gamma\tau + 4\gamma^2\tau^2}}{8b^2} > 0$$

because $16[a-c-\tilde{\mu}(x)\tilde{q}(x)]^2 b^2 > 0$ and

$$Sol_2 = \frac{4[a-c-\tilde{\mu}(x)\tilde{q}(x)]b + 2\gamma\tau + \sqrt{16[a-c-\tilde{\mu}(x)\tilde{q}(x)]b\gamma\tau + 4\gamma^2\tau^2}}{8b^2} > 0$$

so that: $Sol_1 < Sol_2$*. Using* Sol_1*, we obtain* $\mu^*(0) = \frac{-2\gamma\tau+\sqrt{16[a-c-\tilde{\mu}(0)\tilde{q}(0)]b\gamma\tau+4\gamma^2\tau^2}}{4b\gamma Sol_1}$ *and* $\check{x}^* = \frac{-2\gamma\tau+\sqrt{16[a-c-\tilde{\mu}(x)\tilde{q}(x)]b\gamma\tau+4\gamma^2\tau^2}}{4b\tau}$*. Note that:* $\mu^*(0) > 0$ *and* $\check{x}^* > 0$ *because* $16\left[a - c - \tilde{\mu}(x)\tilde{q}(x)\right] b\gamma\tau > 0$*. However, plugging* Sol_2 *in* $\mu^*(0)$ *and* $\check{x}^*$ *leads to* $\mu^*(0) = \frac{-2\gamma\tau-\sqrt{16[a-c-\tilde{\mu}(0)\tilde{q}(0)]b\gamma\tau+4\gamma^2\tau^2}}{4b\gamma Sol_2} < 0$ *and* $\check{x}^* = \frac{-2\gamma\tau-\sqrt{16[a-c-\tilde{\mu}(x)\tilde{q}(x)]b\gamma\tau+4\gamma^2\tau^2}}{4b\tau} < 0$*. This implies that the unique solution for* Γ^* *is:*

$$q^* = \frac{4\left[a - c - \tilde{\mu}(x)\tilde{q}(x)\right] b + 2\gamma\tau - \sqrt{16\left[a - c - \tilde{\mu}(x)\tilde{q}(x)\right] b\gamma\tau + 4\gamma^2\tau^2}}{8b^2}$$

and $\pi^* = b(q^*)^2 - f$*.*[14] *If* $\check{x}^* \geq X$ *then the support of* μ^* *is* $[0, X]$ *and integrating* μ^* *on the new support gives* $q^* = \frac{2[a-c-\tilde{\mu}(X)\tilde{q}(X)]X-\tau X^2}{2(\gamma+2bX)}$*.*[15] $\diamond$

Proof 2 *Previously, note that to determine the effect of each parameter on the equilibrium distribution can be cumbersome. In what follows, we perform a simpler analysis. From Proof 1, we know that the equilibrium profit is given by the following expression* $[a - c - \tilde{\mu}(x)\tilde{q}(x) - 2bq]^2 = 2\gamma\tau q$*. Let* g_1 *be a continuous function on* $\mathbb{R}_+$ *defined as* $g_1(q) = \left[a - c - \tilde{\mu}(x)\tilde{q}(x) - 2bq\right]^2$ *and observe that* $g_1(0) = \left[a - c - \tilde{\mu}(x)\tilde{q}(x)\right]^2 > 0$*,* $\lim\limits_{q\to+\infty} g_1(q) = +\infty$ *and* $\frac{\partial g_1(q)}{\partial q} = -4b\left[a - c - \tilde{\mu}(x)\tilde{q}(x) - 2\right.$
Let g_2 *be a continuous function on* $\mathbb{R}_+$ *defined as* $g_2(q) = 2\gamma\tau q$ *and note that* $g_2(0) = 0$*,* $\lim\limits_{q\to+\infty} g_2(q) = +\infty$ *and* $\frac{\partial g_2(q)}{\partial \Gamma} > 0$*. For the clarity of the exposition, we plot* g_1 *(blue line),* g_2 *(red line) and the locus of the equilibrium profit* q^* *on a graph (see Figure 5). Afterward, we compute the derivative of these two functions with respect to each parameter to determine the effect on* q^* *and* π^**. Then, we compute the derivative of the trade gradient* tg^* *with respect to each parameter. Since* $\int_{Supp(\mu^*)} \mu^*(x)dx = 1$*, when the slope of the trade gradient decreases (respectively increases), the threshold country* $\check{x}^*$ *increases (respectively decreases) and the density of firms in* $x = 0$ *decreases (respectively increases).*[16]

- **Effect of parameter** a**:** $\frac{\partial g_1(0)}{\partial a} > 0$ *(because* $a - c > 0$*),* $\frac{\partial^2 g_1(q)}{\partial a \partial q} < 0$*,* $\frac{\partial g_2(q)}{\partial a} = 0$ *and* $\frac{\partial^2 g_2(q)}{\partial a \partial q} = 0$*. This leads to:* $\frac{\partial q^*}{\partial a} > 0$ *and so* $\frac{\partial \pi^*}{\partial a} > 0$*,* $\frac{\partial tg^*}{\partial a} < 0$*,* $\frac{\partial \mu^*(0)}{\partial a} < 0$ *and* $\frac{\partial \check{x}^*}{\partial a} > 0$*.*

- **Effect of parameter** c**:** $\frac{\partial g_1(0)}{\partial c} < 0$ *(because* $a - c > 0$*),* $\frac{\partial^2 g_1(q)}{\partial c \partial q} > 0$*,* $\frac{\partial g_2(0)}{\partial c} = 0$ *and* $\frac{\partial^2 g_2(q)}{\partial c \partial q} = 0$*. This leads to:* $\frac{\partial q^*}{\partial c} < 0$ *and so* $\frac{\partial \pi^*}{\partial c} < 0$*,* $\frac{\partial tg^*}{\partial c} > 0$*,* $\frac{\partial \mu^*(0)}{\partial c} > 0$ *and* $\frac{\partial \check{x}^*}{\partial c} < 0$*.*

- **Effect of parameter** f**:** *trivial.* $\diamond$

Proof 3 *From Proposition 1, we have: for* $t = 1$*,*

$$\mu_1^*(x) = \frac{a - c - \tilde{\mu}(x)\tilde{q}(x) - \tau x}{\gamma q_1^*} - \frac{2b}{\gamma} \qquad \textit{with} \qquad q_1^* = \sqrt{\frac{\pi_1^* + f}{b}}$$

[14] We assume that $\pi^* > 0$.
[15] Both cases exhibit similar results. Therefore, for the clarity of the exposition, we only display results for $\check{x}^* < X$ in the article.
[16] We perform the case of a, c and f. It is easy to use the same method for γ, b and τ.

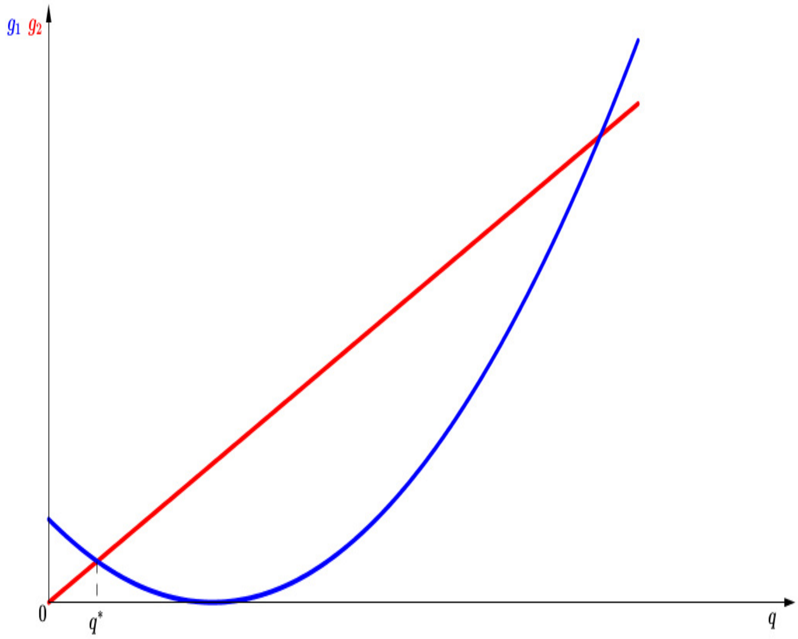

Figure .1 – Equilibrium q^*

with and . For $t = 2$, by solving (25), we obtain:

$$\mu_2^*(x) = \frac{a - c - \tilde{\mu}(x)\tilde{q}(x) - \tau x}{\gamma\sqrt{\frac{\pi_2^*+f}{b}}} - \frac{2b}{\gamma} - (1-\rho)\mu_1^*(x)$$

After simple algebra, we find:

$$\mu_2^*(x) = \frac{a - c - \tilde{\mu}(x)\tilde{q}(x) - \tau x}{\gamma q_2^*} - \frac{2b\rho}{\gamma}$$

with

$$q_2^* = \frac{\sqrt{\frac{\pi_2^*+f}{b}}\sqrt{\frac{\pi_1^*+f}{b}}}{\sqrt{\frac{\pi_1^*+f}{b}} - (1-\rho)\sqrt{\frac{\pi_2^*+f}{b}}}$$

Following Proof 1, the analytical solution for q_2^* is:

$$q_2^* = \frac{4\left[a - c - \tilde{\mu}(x)\tilde{q}(x)\right]b\rho + 2\gamma\tau - \sqrt{16\left[a - c - \tilde{\mu}(x)\tilde{q}(x)\right]b\rho\gamma\tau + 4\gamma^2\tau^2}}{8b^2\rho^2}$$

For $t = 3$, by solving (25) again, we get:

$$\mu_3^*(x) = \frac{a - c - \tilde{\mu}(x)\tilde{q}(x) - \tau x}{\gamma\sqrt{\frac{\pi_3^*+f}{b}}} - \frac{2b\rho}{\gamma} - (1-\rho)\mu_2^*(x) - (1-\rho)^2\mu_1^*(x)$$

After simple algebra, we find:

$$\mu_3^*(x) = \frac{a - c - \tilde{\mu}(x)\tilde{q}(x) - \tau x}{\gamma q_3^*} - \frac{2b\rho}{\gamma}$$

with

$$q_3^* = \frac{\sqrt{\frac{\pi_3^*+f}{b}}\sqrt{\frac{\pi_2^*+f}{b}}}{\sqrt{\frac{\pi_2^*+f}{b}} - (1-\rho)\sqrt{\frac{\pi_3^*+f}{b}}}$$

Following Proof 1 again, the analytical solution for q_3^* is:

$$q_3^* = \frac{4\left[a - c - \tilde{\mu}(x)\tilde{q}(x)\right]b\rho + 2\gamma\tau - \sqrt{16\left[a - c - \tilde{\mu}(x)\tilde{q}(x)\right]b\rho\gamma\tau + 4\gamma^2\tau^2}}{8b^2\rho^2}$$

By iteration, it is easy to show that: for $t > 1$,

$$\mu^*(x) = \frac{a - c - \tilde{\mu}(x)\tilde{q}(x) - \tau x}{\gamma q^*} - \frac{2b\rho}{\gamma}$$

with

$$q^* = \frac{4\left[a - c - \tilde{\mu}(x)\tilde{q}(x)\right]b\rho + 2\gamma\tau - \sqrt{16\left[a - c - \tilde{\mu}(x)\tilde{q}(x)\right]b\rho\gamma\tau + 4\gamma^2\tau^2}}{8b^2\rho^2}$$

Let us now determine the value of (instantaneous) profits π_t for $t > 1$. By iteration, we have:

$$q^* = \frac{\sqrt{\frac{\pi_t^*+f}{b}}\sqrt{\frac{\pi_{t-1}^*+f}{b}}}{\sqrt{\frac{\pi_{t-1}^*+f}{b}} - (1-\rho)\sqrt{\frac{\pi_t^*+f}{b}}}$$

By iteration, we find:

$$\pi_t^* = b\left(\frac{q_1^*}{\sum_{k=0}^{t-2}(1-\rho)^k q_1^* + (1-\rho)^t}\right)^2 - f$$

which is fairly complex. However, after some simple computations, one can find that $\pi_{t+1}^* < \pi_t^*$ if:

$$q_1^* > \rho$$

◇